

# Meta-Stable Brane Configurations of Multiple Product Gauge Groups with Orientifold 6 Plane

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## Abstract

Starting from an  $\mathcal{N} = 1$  supersymmetric electric gauge theory with the gauge group  $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  with fundamentals for each gauge group, the bifundamentals, a symmetric flavor and a conjugate symmetric flavor for  $SU(N_c)$ , we apply Seiberg dual to each gauge group, obtain the  $\mathcal{N} = 1$  supersymmetric dual magnetic gauge theories with dual matters including the gauge singlets, and describe the intersecting brane configurations of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of this gauge theory. We also discuss the case where a symmetric flavor is replaced by an antisymmetric flavor. Next we apply to the case for  $\mathcal{N} = 1$  supersymmetric electric gauge theory with the gauge group  $SO(N_c) \times SU(N'_c) \times SU(N''_c)$  with flavors for each gauge group and the bifundamentals. Finally, we describe the case where the orientifold 6-plane charge is reversed.

# 1 Introduction

The nonsupersymmetric meta-stable vacua exist in  $\mathcal{N} = 1$  super QCD with massive fundamental quarks where the classical flat directions can be lifted by quantum corrections which generate positive mass terms for the pseudomoduli [1]. See also the review paper [2] for the recent developments of dynamical supersymmetry breaking. Turning on the quark masses in the electric theory corresponds to deform the magnetic superpotential by adding a linear term of a singlet field in the dual magnetic theory. The misalignment of D4-branes(01236) connecting NS5'-brane(012389) in the type IIA string theory, corresponding to magnetic theory, can be analyzed as the nontrivial F-term conditions providing nonzero vacuum expectation values of dual quarks and some of D4-branes connecting NS5'-brane can move in other two directions freely. See the review paper [3] for the gauge theory and the brane dynamics.

When an orientifold 6-plane(0123789) is added into the standard brane configuration for a single unitary gauge group, consisting of a NS5-brane(012345), a NS5'-brane, D4-branes and D6-branes(0123789), there exist two possible brane configurations. Either NS-brane overlaps with an orientifold 6-plane at  $x^6 = 0$  or there is no overlap between NS-brane and an orientifold 6-plane at  $x^6 = 0$ . The former has three NS-branes and the gauge theory is described by [4, 5, 6, 7] or [8, 9, 10, 11] depending on whether the NS5-brane overlaps with O6-plane or the NS5'-brane overlaps with O6-plane. The latter has two NS-branes and the gauge theory and corresponding brane configuration are studied by [12].

As an orientifold 6-plane is added into the standard brane configuration for the product of two unitary gauge groups [4, 13, 14], consisting of three NS-branes, D4-branes and D6-branes, there exist also two possible brane configurations. Either five NS-branes where an O6-plane has common value at  $x^6 = 0$  with NS-brane or four NS-branes where at  $x^6 = 0$  there is no NS-brane. The former brane configuration is described by [15] while the latter brane configuration is described by [16, 17].

When we add an orientifold 6-plane to the standard brane configuration for the product of three unitary gauge groups [13, 18], consisting of four NS-branes, D4-branes and D6-branes, there exist also two possible brane configurations. Either seven NS-branes where an O6-plane has common value at  $x^6 = 0$  with NS-brane or six NS-branes where at  $x^6 = 0$  there is no NS-brane. We'll study these brane configurations in details in the context of nonsupersymmetric meta-stable brane configurations.

In this paper, we continue to study for the meta-stable brane configurations of type IIA string theory in the context of three(and multiple) product gauge groups in the presence of an orientifold 6-plane. When the total number of NS-branes is seven, the gauge group is described

by the product of three unitary gauge groups while for the case where the total number of NS-branes is six, one of the gauge groups contains orthogonal or symplectic gauge group as well as two unitary gauge groups, depending whether O6-plane charge is positive or negative. In particular, the Seiberg dual for the middle gauge group has a new feature that there exist two possible magnetic brane configurations as in [18]. Each of them has the different magnetic superpotential and contains the different matter contents. One can generalize the meta-stable brane configurations to the ones corresponding to a multiple product of gauge groups. Then one takes the Seiberg dual for the first gauge group factor, the last gauge group factor or for any gauge group factor except the first and last gauge group factors. One can write down the magnetic superpotentials in terms of the interactions between the gauge singlets and dual matters.

In section 2, we describe the type IIA brane configuration corresponding to the electric theory based on the  $\mathcal{N} = 1$   $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge theory with fundamentals, bifundamentals, a symmetric flavor and a conjugate symmetric flavor for  $SU(N_c)$ , and deform this theory by adding the mass term for the quarks for each gauge group. Then we construct the Seiberg dual magnetic theories for each gauge group with corresponding dual matters as well as additional gauge singlets. After that, the nonsupersymmetric brane configurations are found by recombination and splitting for the flavor D4-branes. One generalizes to the meta-stable brane configurations corresponding to a multiple product of gauge groups and describe them very briefly.

In section 3, we consider the type IIA brane configuration corresponding to the electric theory based on the  $\mathcal{N} = 1$   $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge theory with fundamentals, bifundamentals, eight-fundamentals, an antisymmetric flavor and a conjugate symmetric flavor for  $SU(N_c)$ , and deform this theory by adding the mass terms for the quarks for each gauge group. We present the Seiberg dual magnetic theories for each gauge group with corresponding dual matters as well as additional gauge singlets. Finally, the nonsupersymmetric brane configurations are found. One also generalizes to the meta-stable brane configurations corresponding to a multiple product of gauge groups.

In section 4, we study the type IIA brane configuration corresponding to the electric theory based on the  $\mathcal{N} = 1$   $SO(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge theory with fundamentals, vectors, and bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. Explicitly we construct the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as extra gauge singlets and the nonsupersymmetric brane configurations are found. The generalization to a multiple product of gauge groups is also described.

In section 5, we explain the type IIA brane configuration corresponding to the electric theory based on the  $\mathcal{N} = 1$   $Sp(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge theory with fundamentals, and bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. After that we describe the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as extra gauge singlets. Finally, the nonsupersymmetric brane configurations are found from the magnetic brane configurations. The generalization to a multiple product of gauge groups is discussed.

Finally, in section 6, we summarize what we have found in this paper and make some comments for the future direction.

## 2 Meta-stable brane configurations of multiple product gauge theories

### 2.1 Electric theory

Let us describe the gauge theory with triple product gauge groups  $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  where the symmetric and a conjugate symmetric tensors are present in addition to the fundamentals and bifundamentals. The matter contents are characterized by

- $N_f$ -chiral multiplets  $Q$  are in the representation  $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$ , and  $N_f$ -chiral multiplets  $\tilde{Q}$  are in the representation  $(\overline{\mathbf{N}}_c, \mathbf{1}, \mathbf{1})$ , under the gauge group
- $N'_f$ -chiral multiplets  $Q'$  are in the representation  $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ , and  $N'_f$ -chiral multiplets  $\tilde{Q}'$  are in the representation  $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{1})$ , under the gauge group
- $N''_f$ -chiral multiplets  $Q''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c)$ , and  $N''_f$ -chiral multiplets  $\tilde{Q}''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}}''_c)$ , under the gauge group
- The flavor-singlet field  $F$  is in the bifundamental representation  $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$ , and its conjugate field  $\tilde{F}$  is in the bifundamental representation  $(\overline{\mathbf{N}}_c, \mathbf{N}'_c, \mathbf{1})$ , under the gauge group
- The flavor-singlet field  $G$  is in the bifundamental representation  $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}}''_c)$ , and its conjugate field  $\tilde{G}$  is in the bifundamental representation  $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{N}''_c)$ , under the gauge group
- The flavor-singlet field  $S$ , which is in a symmetric tensor representation under the  $SU(N_c)$ , is in the representation  $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1}, \mathbf{1})$ , and its conjugate field  $\tilde{S}$  is in the representation  $(\frac{1}{2}\overline{\mathbf{N}}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1}, \mathbf{1})$ , under the gauge group

If we ignore the symmetric and conjugate symmetric tensors  $S$  and  $\tilde{S}$ , this theory was studied in [13, 18]. If we put to  $Q'', \tilde{Q}'', G, \tilde{G}, S$  and  $\tilde{S}$  zero, then this becomes the product gauge group theory with fundamentals and bifundamentals [4, 14, 13, 19]. Moreover, if we put to  $Q', \tilde{Q}', Q'', \tilde{Q}'', F, \tilde{F}, G$ , and  $\tilde{G}$  zero, then this theory is described by a single gauge group with fundamentals, a symmetric tensor, and a conjugate symmetric tensor [4, 5, 6, 7].

Now it is easy to check from the matter contents above that the coefficient of the beta function of the first gauge group is given by  $b_{SU(N_c)} = 3N_c - N_f - N'_c - (N_c + 2)$  where the index of the symmetric representation of  $SU(N_c)$  corresponding to  $S$  and  $\tilde{S}$  is equal to  $\frac{1}{2}(N_c + 2)$ . On the other hand, the coefficient of the beta function of the second gauge group is given by  $b_{SU(N'_c)} = 3N'_c - N'_f - N_c - N''_c$ . Moreover, the coefficient of the beta function of the third gauge group is given by  $b_{SU(N''_c)} = 3N''_c - N''_f - N'_c$ . This theory is asymptotically free when the condition  $b_{SU(N_c)} > 0$  is satisfied for the  $SU(N_c)$  gauge group, when the condition  $b_{SU(N'_c)} > 0$  is satisfied for the  $SU(N'_c)$  gauge group, and when the condition  $b_{SU(N''_c)} > 0$  is satisfied for the  $SU(N''_c)$  gauge group. We'll see how these coefficients change in the magnetic theory. We denote the strong coupling scales for  $SU(N_c)$  as  $\Lambda_1$ , for  $SU(N'_c)$  as  $\Lambda_2$  and for  $SU(N''_c)$  as  $\Lambda_3$  respectively.

The electric superpotential by adding the mass terms for  $Q, Q'$  and  $Q''$  is given by

$$W_{elec} = \left( \mu A^2 + QA\tilde{Q} + SA\tilde{S} + \tilde{F}AF + \mu' A'^2 + Q'A'\tilde{Q}' + \tilde{F}A'F + \tilde{G}A'G \right. \\ \left. + \mu'' A''^2 + Q''A''\tilde{Q}'' + \tilde{G}A''G \right) + mQ\tilde{Q} + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''.$$

After integrating the adjoint fields  $A$  for  $SU(N_c)$ ,  $A'$  for  $SU(N'_c)$  and  $A''$  for  $SU(N''_c)$  and taking  $\mu, \mu'$  and  $\mu''$  to infinity limit which is equivalent to take any two NS-branes be perpendicular to each other (in other words, the nearest NS-branes for given  $NS5$ -brane are  $NS5'$ -branes while those for  $NS5'$ -brane are  $NS5$ -branes), the mass-deformed electric superpotential becomes  $W_{elec} = mQ\tilde{Q} + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''$ .

The type IIA brane configuration for this mass-deformed theory can be described by as follows. We introduce the two complex coordinates

$$v \equiv x^4 + ix^5 \quad \text{and} \quad w \equiv x^8 + ix^9$$

for convenience. The  $N_c$ -color D4-branes (01236) are suspended between the  $NS5_M$ -brane (012345) located at  $x^6 = 0$  and the  $NS5'_L$ -brane (012389) along positive  $x^6$  direction, together with  $N_f$  D6-branes (0123789) which are parallel to  $NS5'_L$ -brane and have nonzero  $v$  direction. The  $NS5$ -brane is located at the right hand side of the  $NS5'_L$ -brane along the positive  $x^6$  direction and there exist  $N'_c$ -color D4-branes suspended between them, with  $N'_f$  D6-branes which have nonzero  $v$  direction. Moreover, the  $NS5'_R$ -brane is located at the right hand side of the  $NS5$ -brane along the positive  $x^6$  direction and there exist  $N''_c$ -color D4-branes suspended between them, with  $N''_f$  D6-branes which have nonzero  $v$  direction. There exists an orientifold 6-plane (0123789) at the origin  $x^6 = 0$  and it acts as  $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ . Then the mirrors of above branes appear in the negative  $x^6$  region and are denoted by bar on the

corresponding branes. From the left to the right, there are  $\overline{NS5'_R}$ -,  $\overline{NS5}$ -,  $\overline{NS5'_L}$ -,  $NS5_M$ -,  $NS5'_L$ -,  $NS5$ -, and  $NS5'_R$ -branes.

We summarize the  $\mathcal{N} = 1$  supersymmetric electric brane configuration in type IIA string theory as follows:

- Three NS5-branes in (012345) directions
- Four NS5'-branes in (012389) directions
- Two sets of  $N_c(N'_c)[N''_c]$ -color D4-branes in (01236) directions
- Two sets of  $N_f(N'_f)[N''_f]$  D6-branes in (0123789) directions
- O6-plane in (0123789) directions with  $x^6 = 0$

Now we draw this electric brane configuration in Figure 1 and we put the coincident  $N_f(N'_f)[N''_f]$  D6-branes with positive  $x^6$  in the nonzero  $v$  direction in general. This brane configuration can be obtained from the brane configuration of [15] by adding the two outer NS5'-branes(i.e.,  $\overline{NS5'_R}$ -brane and  $NS5'_L$ -brane), two sets of  $N''_c$  D4-branes and two sets of  $N''_f$  D6-branes or from the one of [13] with the gauge theory of triple product gauge groups by adding O6-plane and the extra NS-branes, D4-branes and D6-branes. Then the mirrors with negative  $x^6$  can be constructed by using the action of O6-plane and are located at the positions by changing (456) directions for original branes with minus signs. The quarks  $Q(Q')[Q'']$  and  $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$  correspond to strings stretching between the  $N_c(N'_c)[N''_c]$ -color D4-branes with  $N_f(N'_f)[N''_f]$  D6-branes. The bifundamentals  $F(G)$  and  $\tilde{F}(\tilde{G})$  correspond to strings stretching between the  $N_c(N'_c)$ -color D4-branes with  $N'_c(N''_c)$ -color D4-branes. The symmetric and a conjugate symmetric tensors  $S$  and  $\tilde{S}$  correspond to strings stretching between  $N_c$  D4-branes with positive  $x^6$  and its mirror  $N_c$  D4-branes with negative  $x^6$ .

## 2.2 Magnetic theory with dual for first gauge group

By brane motion [3], one gets the Seiberg dual theory for the gauge group  $SU(N_c)$  and dualized gauge group's dynamical scale is far above that of the gauge groups  $SU(N'_c)$  and  $SU(N''_c)$ . From the magnetic brane configuration which is shown in Figure 2A, the linking number [20] of  $NS5'_L$ -brane(which is the mirror of  $NS5'_L$ -brane in an electric theory) can be computed and is  $L_5 = \frac{N_f}{2} - \tilde{N}_c + N_f + N'_c$  as in the situation of [7]. On the other hand, the linking number of  $\overline{NS5'_L}$ -brane from the electric brane configuration in Figure 1 can be read off and is given by  $L_5 = -\frac{N_f}{2} + N_c - N'_c$ . Then the number of dual color  $\tilde{N}_c$ , by linking number conservation, is given by

$$\tilde{N}_c = 2(N_f + N'_c) - N_c.$$

Let us draw this magnetic brane configuration in Figure 2A and recall that we put the

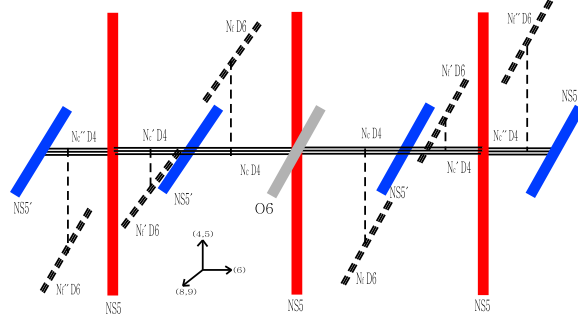


Figure 1: The  $\mathcal{N} = 1$  supersymmetric electric brane configuration with  $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(Q')[Q'']$  and  $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(G), \tilde{F}(\tilde{G})$ , a symmetric tensor  $S$ , and a conjugate symmetric tensor  $\tilde{S}$ . The O6-plane is located at the origin  $x^6 = 0$ . The two  $NS5'_L$ -branes with positive  $x^6$  coordinates are denoted by  $NS5'_{L,R}$ -branes. The mirror branes located at the negative region of  $x^6$  are preserved under the O6-plane action. The mass terms for the quarks are realized geometrically by the displacement of D6-branes to the  $v$  direction.

coincident  $N_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q'$  and  $Q''$  by putting  $N'_f$  and  $N''_f$  D6-branes at  $v = 0$ . The  $N_f$  created D4-branes connecting between D6-branes and  $NS5'_L$ -brane can move freely in the  $w$ -direction. Moreover, since  $N'_c$  or  $N''_c$  D4-branes are suspending between two  $NS5'_{L,R}$ -branes located at different  $x^6$  coordinate, these D4-branes can slide along the  $w$ -direction also. If we ignore the NS5-brane,  $N'_c$  D4-branes,  $N'_f$  D6-branes, the  $NS5'_R$ -brane,  $N''_c$  D4-branes and  $N''_f$  D6-branes(detaching these branes from Figure 2A), then this brane configuration leads to the  $\mathcal{N} = 1$  magnetic theory with gauge group  $SU(\tilde{N}_c = 2N_f - N_c)$  with  $N_f$  massive fundamental flavors plus symmetric, conjugate symmetric flavors and gauge singlets [21, 7]. On the other hand, when we ignore the  $NS5'_R$ -brane,  $N'_c$  D4-branes and  $N'_f$  D6-branes(detaching these branes from Figure 2A), then this brane configuration leads to the  $\mathcal{N} = 1$  magnetic theory with gauge group  $SU(\tilde{N}_c = 2N_f + 2N'_c - N_c) \times SU(N'_c)$  with fundamental flavors, bifundamentals, symmetric, conjugate symmetric flavors and gauge singlets in Figure 4 of [15].

Now let us recombine  $\tilde{N}_c$  flavor D4-branes among  $N_f$  flavor D4-branes(connecting between D6-branes and  $NS5'_L$ -brane) with the same number of color D4-branes(connecting between  $NS5_M$ -brane and  $NS5'_L$ -brane) and push them in  $+v$  direction from Figure 2A. We assume that  $N_c \geq N_f + 2N'_c$ . After this procedure, there are no color D4-branes between  $NS5_M$ -brane and  $NS5'_L$ -brane. For the flavor D4-branes, we are left with only  $(N_f - \tilde{N}_c) = N_c - N_f - 2N'_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane in Figure 2B.

Then the dual magnetic gauge group is  $SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$  and the matter contents are as follows:





- The  $N_c'^2$ -fields  $\Phi'$  is in the representation  $(\mathbf{1}, \mathbf{N}_c'^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$  under the gauge group
- Moreover, there are additional  $N_f(2N_f + 1)$  gauge-singlets
- $N_f^2$ -fields  $N$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$  under the gauge group
- $\frac{1}{2}N_f(N_f + 1)$ -fields  $P$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ , and its conjugate  $\frac{1}{2}N_f(N_f + 1)$ -fields  $\tilde{P}$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ , under the gauge group

The additional  $N_f$ - $SU(N_c')$  fundamentals  $X'$  and  $N_f$ - $SU(N_c')$  antifundamentals  $\tilde{X}'$  are originating from the  $SU(N_c)$  chiral mesons  $\tilde{F}Q$  and  $F\tilde{Q}$  respectively. Therefore, there are free indices for a single color and a single flavor. Then the strings stretching between the  $N_f$  D6-branes and  $N_c'$  D4-branes will give rise to these additional  $N_f$ - $SU(N_c')$  fundamentals and  $N_f$ - $SU(N_c')$  antifundamentals. The gauge singlet  $M$  corresponds to the  $SU(N_c)$  chiral meson  $Q\tilde{Q}$  where the color indices are contracted. Because the  $N_f$  D6-branes are parallel to the  $NS5'_L$ -brane from Figure 2A, the newly created  $N_f$ -flavor D4-branes can slide along the plane consisting of these  $N_f$  D6-branes and  $NS5'_L$ -brane freely. The fluctuations of the gauge-singlet  $M$  correspond to the motion of  $N_f$  flavor D4-branes along (789) directions in Figure 2A. As we will see later, for the nonsupersymmetric brane configuration, a misalignment for the  $N_f$ -flavor D4-branes arises and some of the vacuum expectation value of  $M$  is fixed and the remaining components are arbitrary. The  $\Phi'$  corresponds to the  $SU(N_c)$  chiral meson  $F\tilde{F}$  where the color indices for the first gauge group are contracted each other. The fluctuations of the singlet  $\Phi'$  correspond to the motion of  $N_c'$  D4-branes suspended two  $NS5'_{L,R}$ -branes along the (789) directions in Figure 2A. Although the gauge singlets  $N, P$  and  $\tilde{P}$  appear in the dual magnetic superpotential for the general rotation angles of NS-branes and D6-branes, the case we are considering does not contain these gauge singlets, as observed in [7].

The coefficient of the beta function of the first dual gauge group factor, as done in electric theory, is given by  $b_{SU(\tilde{N}_c)}^{mag} = 3\tilde{N}_c - N_f - N_c' - (\tilde{N}_c + 2)$  and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(N_c')}^{mag} = 3N_c' - N_f' - \tilde{N}_c - N_c'' - N_f - N_c'$ . Finally, the coefficient of the beta function of the third dual gauge group factor is  $b_{SU(N_c'')}^{mag} = 3N_c'' - N_f'' - N_c' = b_{SU(N_c'')}$ . Then both  $SU(\tilde{N}_c)$ ,  $SU(N_c')$ , and  $SU(N_c'')$  gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region with  $b_{SU(\tilde{N}_c)}^{mag} < 0$ ,  $b_{SU(N_c')}^{mag} < 0$ , and  $b_{SU(N_c'')}^{mag} < 0$ .

The dual magnetic superpotential coming from [7] is given by

$$W_{dual} = (Mq\tilde{s}s\tilde{q} + mM) + \tilde{f}X'q + f\tilde{q}\tilde{X}' + \Phi'f\tilde{f}.$$

Then,  $q\tilde{s}s\tilde{q}$  has rank  $\tilde{N}_c$  while  $m$  has a rank  $N_f$ . Therefore, the derivative of the superpotential  $W_{dual}$  with respect to  $M$ , cannot be satisfied if the rank  $N_f$  exceeds  $\tilde{N}_c$  and the supersymmetry

is broken. The classical moduli space of vacua can be obtained from F-term equations and one gets [7]

$$\begin{aligned}
q\tilde{s}s\tilde{q} + m &= 0, & \tilde{s}s\tilde{q}M + \tilde{f}X' &= 0, \\
s\tilde{q}Mq &= 0, & \tilde{q}Mq\tilde{s} &= 0, \\
Mq\tilde{s}s + \tilde{X}'f &= 0, & X'q + \Phi'f &= 0, \\
q\tilde{f} &= 0, & \tilde{q}\tilde{X}' + \tilde{f}\Phi' &= 0, \\
f\tilde{q} &= 0, & f\tilde{f} &= 0.
\end{aligned}$$

Some of F-term equations are satisfied if one takes the zero vacuum expectation values for the fields  $f, \tilde{f}, X'$  and  $\tilde{X}'$ .

Then the gauge group and matter contents we consider are summarized as follows:

gauge group :	$SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$
matter :	$ \begin{array}{ll} q_f \oplus \tilde{q}_{\tilde{f}} & (\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f) \\ Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'} & (\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f) \\ Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''} & (\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f) \\ f \oplus \tilde{f} & (\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1}) \\ G \oplus \tilde{G} & (\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square) \\ s \oplus \tilde{s} & (\mathbf{symm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{symm}}, \mathbf{1}, \mathbf{1}) \\ (X'_n \equiv) \tilde{F}Q \oplus F\tilde{Q} (\equiv \tilde{X}'_n) & (\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n, \tilde{n} = 1, \dots, N_f) \\ (M_{f,\tilde{g}} \equiv) Q\tilde{Q} & (\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{g} = 1, \dots, N_f) \\ (\Phi' \equiv) F\tilde{F} & (\mathbf{1}, \mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \\ (P_{f,g} \equiv) Q\tilde{S}Q \oplus \tilde{Q}S\tilde{Q} (\equiv \tilde{P}_{\tilde{f},\tilde{g}}) & (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f}, g, \tilde{g} = 1, \dots, N_f) \\ (N_{f,\tilde{g}} \equiv) Q\tilde{S}S\tilde{Q} & (\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{g} = 1, \dots, N_f) \end{array} $

Then, it is easy to see that  $s\tilde{q}M = 0 = Mq\tilde{s}, q\tilde{s}s\tilde{q} + m = 0$ . Then the solutions can be written as

$$\begin{aligned}
\langle q\tilde{s} \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \langle s\tilde{q} \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{pmatrix}, \\
\langle f \rangle &= \langle \tilde{f} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0.
\end{aligned}$$

Let us expand around a point on the vacua, as done in [1]. Then the remaining relevant terms of superpotential are given by  $W_{dual}^{rel} = M_0 \left( \delta\hat{\varphi} \delta\hat{\tilde{\varphi}} + m \right) + \delta Z \delta\hat{\varphi} s_0 \tilde{q}_0 + \delta\tilde{Z} q_0 \tilde{s}_0 \delta\hat{\tilde{\varphi}}$

by following the fluctuations for the various fields in [7]. Note that there exist three kinds of terms, the vacuum  $\langle q \rangle$  multiplied by  $\delta\tilde{f}\delta X'$ , the vacuum  $\langle \tilde{q} \rangle$  multiplied by  $\delta\tilde{X}'\delta f$ , and the vacuum  $\langle \Phi' \rangle$  multiplied by  $\delta f\delta\tilde{f}$ . By redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [7], the defining function  $\mathcal{F}(v^2)$  [22] can be computed and  $m_{M_0}^2$  will contain  $(\log 4 - 1) > 0$  implying that these are stable.

The minimal energy supersymmetry breaking brane configuration is shown in Figure 2B. If we ignore the NS5-brane,  $N'_c$  D4-branes,  $N'_f$  D6-branes,  $NS5'_R$ -brane,  $N''_c$  D4-branes and  $N''_f$  D6-branes (detaching these from Figure 2B), as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the  $\mathcal{N} = 1$  SQCD with the magnetic gauge group  $SU(\tilde{N}_c = 2N_f - N_c)$  with  $N_f$  massive flavors [21, 7].

The nonsupersymmetric minimal energy brane configuration Figure 2B with a replacement  $N_f$  D6-branes by the NS5'-brane (neglecting the  $NS5'_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes and  $N'_f$  D6-branes) leads to the Figure 5B of [23] with a rotation of NS5'-brane by  $\frac{\pi}{2}$  angle.

In [24, 5], the Riemann surface describing a set of NS-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f+N'_f+N''_f}v^{2N'_f+2N''_f+4}(v^2 - m^2)^{N_f}$  was found. Since we are dealing with seven NS-branes, the magnetic M5-brane configuration in Figure 2 with equal mass for  $q$  and massless for  $Q'(Q'')$  can be characterized by the following seventh order equation for  $t$  as follows:

$$\begin{aligned} & t^7 + \left[ v^{N''_c} \right] t^6 + \left[ v^{N'_c+N''_f} \right] t^5 + \left[ v^{\tilde{N}_c+2N''_f+N'_f}(v-m)^{N_f} \right] t^4 \\ & + \left[ (-1)^{\tilde{N}_c} v^{\tilde{N}_c+3N''_f+2N'_f+2}(v-m)^{2N_f} \right] t^3 + \left[ (-1)^{N'_c} v^{N'_c+4N''_f+3N'_f+6}(v-m)^{3N_f} \right] t^2 \\ & + \left[ (-1)^{N_f+N'_f+N''_c} v^{10+5N'_f+5N''_f+N''_c}(v-m)^{4N_f}(v+m)^{N_f} \right] t \\ & + \left[ (-1)^{N''_f} v^{14+7N'_f+7N''_f}(v-m)^{5N_f}(v+m)^{2N_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Now the asymptotic regions of various NS-branes can be determined by reading off the first two terms of the seventh order curve above giving the  $\overline{NS5'_R}$ -brane asymptotic region, next two terms giving the  $\overline{NS5}$ -brane asymptotic region, next two terms giving the  $\overline{NS5'_L}$ -brane asymptotic region, next two terms giving  $NS5_M$ -brane asymptotic region, next two terms giving  $NS5'_L$ -brane asymptotic region, next two terms giving NS5-brane asymptotic region, and final two terms giving  $NS5'_R$ -brane asymptotic region. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{N'_c+N''_f-N'_c} + \dots & \overline{NS5} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{N''_f+N_f+N'_f+2} + \dots & NS5_M \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{N''_f+2N_f+2N'_f-N'_c+N''_c+4} + \dots & NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow -m, & y &\sim w^{\tilde{N}_c-N'_c+N''_f+N_f+N'_f} + \dots & \overline{NS5'_L} \text{ asymptotic region,} \\ v &\rightarrow -m, & y &\sim w^{N''_c} + \dots & \overline{NS5'_R} \text{ asymptotic region,} \\ v &\rightarrow +m, & y &\sim w^{-\tilde{N}_c+N'_c+N''_f+N_f+N'_f+4} + \dots & NS5'_L \text{ asymptotic region,} \\ v &\rightarrow +m, & y &\sim w^{-N''_c+2N''_f+2N_f+2N'_f+4} + \dots & NS5'_R \text{ asymptotic region} \end{aligned}$$

where we denote the mirror branes by writing the bar on the corresponding NS-brane. The two  $NS5'_{L,R}$ -branes are moving in the  $+v$  direction respectively holding everything else fixed instead of moving D6-branes in the  $+v$  direction [25, 26]. The corresponding mirrors of D4-branes are moved appropriately. The harmonic function in the Tau-NUT space, sourced by  $2N_f$  D6-branes, O6-plane,  $2N'_f$  D6-branes and  $2N''_f$  D6-branes, can be determined once we fix the  $x^6$  position for these branes. Then the first order differential equation for the  $g(s)$  where the absolute value of  $g(s)$  is equal to the absolute value of  $w$  can be solved exactly with the appropriate boundary conditions on  $NS5'_L$  or  $NS5'_R$  asymptotic region from above. Since the extra terms in the harmonic function contribute to the  $g(s)$  as a multiplication factor, the contradiction with the correct statement that  $y$  should vanish only if  $v = 0$  implies that there exists the instability from a new M5-brane mode at some point from the transition of SQCD-like theory description to M-theory description.

## 2.3 Magnetic theory with dual for second gauge group

By moving the NS5-brane in Figure 1 to the left all the way past the  $NS5'_L$ -brane, one arrives at the Figure 3A. The linking number of NS5-brane from Figure 3A is  $L_5 = -\frac{N'_f}{2} + \tilde{N}'_c - N_c$  while the linking number of NS5-brane from Figure 1 is  $L_5 = \frac{N'_f}{2} + N''_c - N'_c$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N''_c + N_c - N'_c.$$

Let us draw this magnetic brane configuration in Figure 3A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless

flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . The  $N'_f$  created D4-branes connecting between D6-branes and  $NS5'_L$ -brane can move freely in the  $w$ -direction. Moreover, since  $N_c$  D4-branes are suspending between the  $NS5_M$ -brane located at the origin  $x^6 = 0$  and  $NS5$ -brane located at different  $x^6$  coordinate, these D4-branes can slide along the  $v$ -direction. Then this brane configuration leads to the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  with fundamentals and bifundamentals, and singlets in Figure 3 of [18].

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and  $NS5'_L$ -brane) with the same number of color D4-branes(connecting between  $NS5$ -brane and  $NS5'_L$ -brane) and push them in  $+v$  direction from Figure 3A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane in Figure 3B.

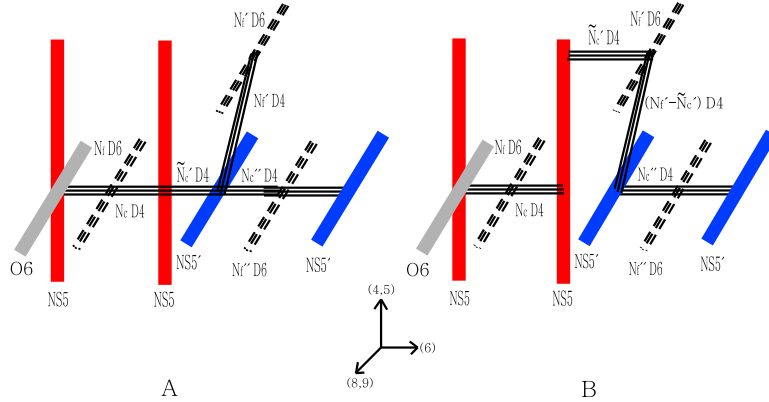


Figure 3: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N''_c + N_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q')[Q'']$  and  $\tilde{Q}(\tilde{q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ ,  $S$  and  $\tilde{S}$ , and gauge singlets in Figure 3A. In Figure 3B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $\tilde{Q}(\tilde{Q}'')$  is given.

The additional  $N'_f$ - $SU(N''_c)$  fundamentals  $X''$  and  $N'_f$ - $SU(N''_c)$  antifundamentals  $\tilde{X}''$  are originating from the  $SU(N'_c)$  chiral mesons  $\tilde{G}Q'$  and  $G\tilde{Q}'$  respectively. Then the strings stretching between the  $N'_f$  D6-branes and  $N''_c$  D4-branes will give rise to these additional  $N'_f$ - $SU(N''_c)$  fundamentals and  $N'_f$ - $SU(N''_c)$  antifundamentals. The gauge singlet  $M'$  corresponds to the  $SU(N'_c)$  chiral meson  $Q'\tilde{Q}'$  where the color indices are contracted. The  $\Phi''$  corresponds to the  $SU(N'_c)$  chiral meson  $G\tilde{G}$  where the color indices for the second gauge group are contracted each other.

The coefficient of the beta function of the first gauge group factor is given by  $b_{SU(N_c)}^{mag} = 3N_c - N_f - \tilde{N}'_c - (N_c + 2)$  and the coefficient of the beta function of the second gauge group

factor is given by  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$ . The coefficient of the beta function of the third gauge group factor is  $b_{SU(N''_c)}^{mag} = 3N''_c - N'_f - \tilde{N}'_c - N'_f - N''_c$ .

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$S \oplus \tilde{S}$	$(\mathbf{symm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{symm}}, \mathbf{1}, \mathbf{1})$
	$(X''_{n'} \equiv) \tilde{G}Q' \oplus G\tilde{Q}' (\equiv \tilde{X}''_{\tilde{n}'})$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi'' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

From the superpotential

$$W_{dual} = (M'q'\tilde{q}' + m'M') + X''\tilde{g}q' + \tilde{X}''\tilde{q}'g + \Phi''g\tilde{g}$$

one sees that  $q'\tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields  $g, \tilde{g}, X''$  and  $\tilde{X}''$ . Then, it is easy to see that  $\tilde{q}'M' = 0 = M'q', q'\tilde{q}' + m' = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'}e^{\phi}\mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'}e^{-\phi}\mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X'' \rangle = \langle \tilde{X}'' \rangle = 0. \end{aligned}$$

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.

The nonsupersymmetric minimal energy brane configuration Figure 3B (neglecting the O6-plane and the mirrors) leads to the Figure 3B of [18] and the Figure 3B with a replacement  $N'_f$  D6-branes by the NS5'-brane (neglecting the  $NS5'_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes and  $N_f$  D6-branes) leads to the Figure 2B of [23] with a rotation of NS5'-brane by  $\frac{\pi}{2}$  angle.

The Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f + N'_f + N''_f} v^{2N_f + 2N''_f + 4} (v^2 - m'^2)^{N'_f}$  was found.

Since we are dealing with seven NS-branes, the magnetic M5-brane configuration in Figure 3 with equal mass for  $q'$  and massless for  $Q(Q'')$  can be characterized by the following seventh order equation for  $t$  as follows:

$$\begin{aligned}
& t^7 + \left[ v^{N_c''} \right] t^6 + \left[ v^{\tilde{N}_c' + N_f''} (v - m')^{N_f'} \right] t^5 + \left[ v^{N_c + 2N_f''} (v - m')^{2N_f'} \right] t^4 \\
& + \left[ (-1)^{N_c} v^{N_c + 3N_f'' + N_c + 2} (v - m')^{3N_f'} \right] t^3 + \left[ (-1)^{\tilde{N}_c'} v^{\tilde{N}_c' + 4N_f'' + 2N_c + 4} (v - m')^{4N_f'} \right] t^2 \\
& + \left[ (-1)^{N_c''} v^{10 + 5N_c + 5N_f'' + N_c''} (v - m')^{5N_f'} \right] t \\
& + \left[ (-1)^{N_c + N_f' + N_f''} v^{14 + 7N_c + 6N_f''} (v - m')^{6N_f'} (v + m')^{N_f'} \right] = 0.
\end{aligned}$$

At nonzero string coupling constant, the NS-branes bend due to their interactions with the D4-branes and D6-branes. Now the asymptotic regions of various NS-branes can be determined similarly. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned}
w & \rightarrow 0, \quad y \sim v^{N_c + N_f'' - \tilde{N}_c' + N_f'} + \dots \quad \overline{NS5} \text{ asymptotic region,} \\
w & \rightarrow 0, \quad y \sim v^{N_c + N_f'' + N_f' + 2} + \dots \quad NS5_M \text{ asymptotic region,} \\
w & \rightarrow 0, \quad y \sim v^{N_f'' + N_f' + \tilde{N}_c' + 2} + \dots \quad NS5 \text{ asymptotic region.}
\end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned}
v & \rightarrow -m', \quad y \sim w^{\tilde{N}_c' - N_c'' + N_f'' + N_f'} + \dots \quad \overline{NS5'_L} \text{ asymptotic region,} \\
v & \rightarrow -m', \quad y \sim w^{N_c''} + \dots \quad \overline{NS5'_R} \text{ asymptotic region,} \\
v & \rightarrow +m', \quad y \sim w^{-\tilde{N}_c' + 3N_c + N_c'' + N_f'' + N_f' + 6} + \dots \quad NS5'_L \text{ asymptotic region,} \\
v & \rightarrow +m', \quad y \sim w^{-N_c'' + 2N_c + N_f'' + 2N_f' + 4} + \dots \quad NS5'_R \text{ asymptotic region.}
\end{aligned}$$

We denote the mirror branes by writing the bar on the corresponding NS-brane. The two  $NS5'_{L,R}$ -branes are moving in the  $+v$  direction respectively holding everything else fixed instead of moving D6-branes in the  $+v$  direction. The corresponding mirrors of D4-branes are moved appropriately.

## 2.4 Magnetic theory with dual for second gauge group

By moving the  $NS5'_L$ -brane in Figure 1 to the right all the way past the NS5-brane, one arrives at the Figure 4A. The linking number of  $NS5'_L$ -brane from Figure 4A is  $L_5 = \frac{N_f'}{2} - \tilde{N}_c' + N_c''$

and the linking number of  $NS5'_L$ -brane from the Figure 1 is  $L_5 = -\frac{N'_f}{2} + N'_c - N_c$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N''_c + N_c - N'_c.$$

Let us draw this magnetic brane configuration in Figure 4A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore the mirror branes and O6-plane(detaching these branes from Figure 4A), then this brane configuration leads to the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  with fundamentals and bifundamentals in Figure 4 of [18]. On the other hand, when we ignore the  $NS5'_R$ -brane,  $N''_c$  D4-branes and  $N''_f$  D6-branes(detaching these branes from Figure 4A), then this brane configuration leads to the  $\mathcal{N} = 1$  magnetic theory with gauge group  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c - N'_c)$  with fundamental flavors, bifundamentals, symmetric, conjugate symmetric flavors and gauge singlets in Figure 5 of [15].

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and NS5-brane) with the same number of color D4-branes(connecting between NS5-brane and  $NS5'_L$ -brane) and push them in  $+v$  direction from Figure 4A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane.

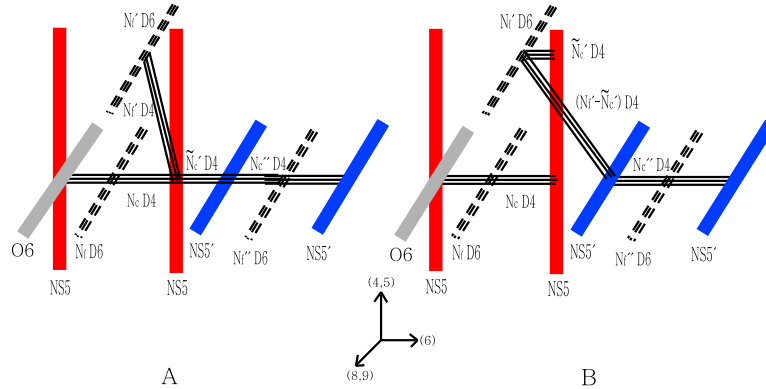


Figure 4: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N''_c + N_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q')[Q'']$  and  $\tilde{Q}(\tilde{q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $f(G)$  and  $\tilde{f}(\tilde{G})$ ,  $S$  and  $\tilde{S}$ , and gauge singlets in Figure 4A. In Figure 4B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $\tilde{Q}(\tilde{Q}'')$  is given.

The additional  $N'_f$ - $SU(N_c)$  fundamentals  $X$  and  $N'_f$ - $SU(N_c)$  antifundamentals  $\tilde{X}$  are originating from the  $SU(N'_c)$  chiral mesons  $FQ'$  and  $\tilde{F}\tilde{Q}'$  respectively. The gauge singlet  $M'$



corresponds to the  $SU(N'_c)$  chiral meson  $Q'\tilde{Q}'$  where the color indices are contracted. The fluctuations of the gauge-singlet  $M'$  correspond to the motion of  $N'_f$  flavor D4-branes along (789) directions in Figure 4B (and their mirrors). The  $\Phi$  corresponds to the  $SU(N'_c)$  chiral meson  $F\tilde{F}$  where the color indices for the second gauge group are contracted each other.

The coefficient of the beta function of the first gauge group factor is given by  $b_{SU(N_c)}^{mag} = 3N_c - N_f - \tilde{N}'_c - (N_c + 2) - N'_f - N_c$ , the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$ , and the coefficient of the beta function of the third gauge group factor is given by  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c$ .

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$S \oplus \tilde{S}$	$(\text{symm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\text{symm}}, \mathbf{1}, \mathbf{1})$
	$(X_{n'} \equiv) FQ' \oplus \tilde{F}\tilde{Q}' (\equiv \tilde{X}_{\tilde{n}'})$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi \equiv) F\tilde{F}$	$(\text{adj}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

From the superpotential

$$W_{dual} = (M'q'\tilde{q}' + m'M') + Xfq' + \tilde{X}\tilde{q}'\tilde{f} + \Phi f\tilde{f}$$

one sees that  $q'\tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $\tilde{q}'M' = 0 = M'q', q'\tilde{q}' + m' = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'}e^{\phi}\mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'}e^{-\phi}\mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X \rangle = \langle \tilde{X} \rangle = 0. \end{aligned}$$

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.

The nonsupersymmetric minimal energy brane configuration Figure 4B (neglecting the O6-plane and the mirrors) leads to the Figure 4B of [18].

The Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f+N'_f+N''_f} v^{2N_f+2N''_f+4} (v^2 - m'^2)^{N'_f}$  was found. Since we are dealing with seven NS-branes, the magnetic M5-brane configuration in Figure 4 with equal mass for  $q'$  and massless for  $Q(Q'')$  can be characterized by the following seventh order equation for  $t$  as follows:

$$\begin{aligned} & t^7 + \left[ v^{N''_c} \right] t^6 + \left[ v^{\tilde{N}'_c+N'_f} \right] t^5 + \left[ v^{N_c+2N''_f+2} \right] t^4 + \left[ (-1)^{N_c} v^{N_c+3N''_f+N_f+6} (v - m')^{N'_f} \right] t^3 \\ & + \left[ (-1)^{\tilde{N}'_c+N_f+N'_f} v^{\tilde{N}'_c+4N''_f+2N_f+10} (v - m')^{2N'_f} (v + m')^{N'_f} \right] t^2 \\ & + \left[ (-1)^{N''_c} v^{16+3N_f+5N''_f+N''_c} (v - m')^{3N'_f} (v + m')^{2N'_f} \right] t \\ & + \left[ (-1)^{N_f+N'_f+N''_f} v^{22+4N_f+7N''_f} (v - m')^{4N'_f} (v + m')^{3N'_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w & \rightarrow 0, \quad y \sim v^{N_c+N''_f+2-\tilde{N}'_c} + \dots \quad \overline{NS5} \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N''_f+N_f+N'_f+4} + \dots \quad NS5_M \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N''_f+N_f+2N'_f-\tilde{N}'_c-N_c+4} + \dots \quad NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v & \rightarrow -m', \quad y \sim w^{\tilde{N}'_c-N''_c+N''_f} + \dots \quad \overline{NS5'_L} \text{ asymptotic region,} \\ v & \rightarrow -m', \quad y \sim w^{N''_c} + \dots \quad \overline{NS5'_R} \text{ asymptotic region,} \\ v & \rightarrow +m', \quad y \sim w^{-\tilde{N}'_c+N''_c+N''_f+N_f+2N'_f+4} + \dots \quad NS5'_L \text{ asymptotic region,} \\ v & \rightarrow +m', \quad y \sim w^{-N''_c+2N''_f+N_f+2N'_f+6} + \dots \quad NS5'_R \text{ asymptotic region.} \end{aligned}$$

Here we denote the mirror branes by writing the bar on the corresponding NS-brane. The two  $NS5'_{L,R}$ -branes are moving in the  $+v$  direction respectively holding everything else fixed instead of moving D6-branes in the  $+v$  direction. The corresponding mirrors of D4-branes are moved appropriately.

## 2.5 Magnetic theory with dual for third gauge group

By moving the NS5-brane with massive  $N''_f$  D6-branes to the right all the way past the  $NS5'_R$ -brane, one arrives at the Figure 5A. The linking number of NS5-brane from Figure 5A is given

by  $L_5 = \frac{N_f''}{2} - \tilde{N}_c''$  and the linking number of NS5-brane from Figure 1 is  $L_5 = -\frac{N_f''}{2} + N_c'' - N_c'$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c'' = N_f'' + N_c' - N_c''.$$

Let us draw this magnetic brane configuration in Figure 5A and recall that we put the coincident  $N_f''$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q'$  by putting  $N_f$  and  $N_f'$  D6-branes at  $v = 0$ . If we ignore the mirror branes and O6-plane(detaching these branes from Figure 5A), then this brane configuration leads to the standard  $\mathcal{N} = 1$  SQCD with the magnetic gauge group  $SU(N_c) \times SU(N_c') \times SU(\tilde{N}_c'' = N_f'' + N_c' - N_c'')$  with fundamentals, bifundamentals, and singlets in Figure 2 of [18].

Now let us recombine  $\tilde{N}_c''$  flavor D4-branes among  $N_f''$  flavor D4-branes(connecting between D6-branes and  $NS5'_R$ -brane) with the same number of color D4-branes(connecting between  $NS5'_R$ -brane and NS5-brane) and push them in  $+v$  direction from Figure 5A. For the flavor D4-branes, we are left with only  $(N_f'' - \tilde{N}_c'') = N_c'' - N_c'$  flavor D4-branes connecting between D6-branes and  $NS5'_R$ -brane.

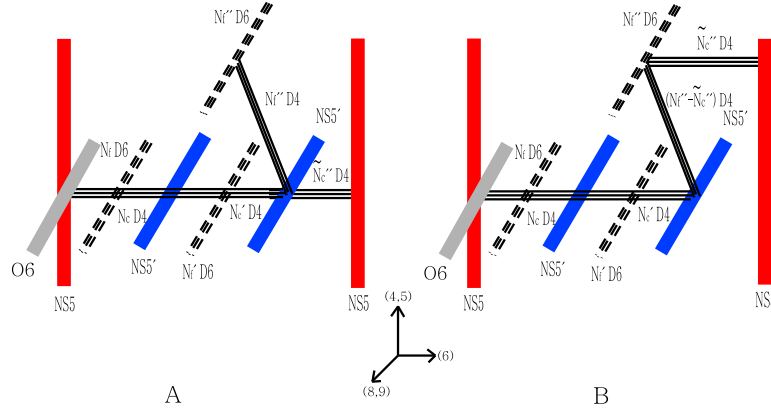


Figure 5: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(N_c') \times SU(\tilde{N}_c'' = N_f'' + N_c' - N_c'')$  gauge group with fundamentals  $Q(Q')[q'']$  and  $\tilde{Q}(\tilde{Q}')[\tilde{q}'']$  for each gauge group, bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ ,  $S$  and  $\tilde{S}$ , and gauge singlets in Figure 5A. In Figure 5B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q')$  and  $\tilde{Q}(\tilde{Q}')$  is given.

The  $N_f''$ - $SU(N_c')$  fundamentals  $X'$  and  $N_f''$ - $SU(N_c')$  antifundamentals  $\tilde{X}'$  are originating from the  $SU(N_c'')$  chiral mesons  $GQ''$  and  $\tilde{G}\tilde{Q}''$  respectively. The gauge singlet  $M''$  corresponds to the  $SU(N_c'')$  chiral meson  $Q''\tilde{Q}''$  where the color indices are contracted. The fluctuations of the gauge-singlet  $M''$  correspond to the motion of  $N_f''$  flavor D4-branes along (789) directions in Figure 5A(and their mirrors). The  $\Phi'$  corresponds to the  $SU(N_c'')$  chiral meson  $G\tilde{G}$  where the color indices for the third gauge group are contracted each other.

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_{f'})$
	$q''_{f''} \oplus \tilde{q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_{f''})$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$S \oplus \tilde{S}$	$(\text{symm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\text{symm}}, \mathbf{1}, \mathbf{1})$
	$(X'_{n''} \equiv) GQ'' \oplus \tilde{G}\tilde{Q}'' (\equiv \tilde{X}'_{\tilde{n}''})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n'', \tilde{n}'' = 1, \dots, N''_{f''})$
	$(M''_{f'', \tilde{g}''} \equiv) Q''\tilde{Q}''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f'', \tilde{g}'' = 1, \dots, N''_{f''})$
	$(\Phi' \equiv) G\tilde{G}$	$(\mathbf{1}, \text{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The coefficient of the beta function of the first gauge group factor  $b_{SU(N_c)}^{mag} = 3N_c - N_f - N'_c - (N_c + 2) = b_{SU(N_c)}$  and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - N_c - \tilde{N}_c'' - N''_f - N'_c$  and the coefficient of the beta function of the third gauge group factor is  $b_{SU(\tilde{N}_c'')}^{mag} = 3\tilde{N}_c'' - N''_f - N'_c$ . Since  $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} > 0$ ,  $SU(N'_c)$  is more asymptotically free than  $SU(N'_c)^{mag}$ .

The superpotential is given by

$$W_{dual} = (M''q''\tilde{q}'' + m''M'') + X'gq'' + \tilde{X}'\tilde{q}''\tilde{g} + \Phi'g\tilde{g}.$$

Then,  $q''\tilde{q}''$  has rank  $\tilde{N}_c''$  while  $m''$  has a rank  $N''_f$ . The derivative of the superpotential  $W_{dual}$  with respect to  $M''$ , cannot be satisfied if the rank  $N''_f$  exceeds  $\tilde{N}_c''$  and the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $\tilde{q}''M'' = 0 = M''q'', q''\tilde{q}'' + m'' = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q'' \rangle &= \begin{pmatrix} \sqrt{m''}e^\phi \mathbf{1}_{\tilde{N}_c''} \\ 0 \end{pmatrix}, \langle \tilde{q}'' \rangle = \begin{pmatrix} \sqrt{m''}e^{-\phi} \mathbf{1}_{\tilde{N}_c''} & 0 \end{pmatrix}, \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M''_0 \mathbf{1}_{N''_f - \tilde{N}_c''} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

One can analyze the one loop computation by expanding the fields around the vacua and it will turn out that states are stable by realizing the mass of  $m_{M''_0}^2$  positive.

The nonsupersymmetric minimal energy brane configuration Figure 5B with a replacement  $N''_f$  D6-branes by the NS5'-brane (neglecting the  $NS5'_L$ -brane,  $N_f$  D6-branes and  $N_c$  D4-branes and  $N'_f$  D6-branes) leads to the Figure 4B of [23].

The Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f+N'_f+N''_f} v^{2N_f+2N'_f+4} (v^2 - m''^2)^{N''_f}$  was found. Since we are dealing with seven NS-branes, the magnetic M5-brane configuration in Figure 5 with equal mass for  $q''$  and massless for  $Q(Q')$  can be characterized by the following seventh order equation for  $t$  as follows:

$$\begin{aligned} & t^7 + \left[ v^{\tilde{N}''_c} \right] t^6 + \left[ v^{N'_c} \right] t^5 + \left[ v^{N_c+N'_f} (v - m'')^{N''_f} \right] t^4 \\ & + \left[ (-1)^{N_c} v^{2N_c+2N'_f+2} (v - m'')^{2N''_f} \right] t^3 + \left[ (-1)^{N_c+N'_c} v^{N'_c+3N_c+3N'_f+6} (v - m'')^{3N''_f} \right] t^2 \\ & + \left[ (-1)^{N'_f+N'_f+\tilde{N}''_c} v^{10+5N_c+4N'_f+\tilde{N}''_c} (v - m'')^{4N''_f} (v + m'')^{N''_f} \right] t \\ & + \left[ (-1)^{N_c} v^{14+5N'_f+7N_c} (v - m'')^{5N''_f} (v + m'')^{2N''_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w & \rightarrow 0, \quad y \sim v^{\tilde{N}''_c} + \dots \quad \overline{NS5} \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N_c+N'_f+N'_f+2} + \dots \quad NS5_M \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{2N'_f+N'_f+2N_c-\tilde{N}''_c+4} + \dots \quad NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v & \rightarrow -m'', \quad y \sim w^{N_c-N'_c+N'_f+N'_f} + \dots \quad \overline{NS5'_L} \text{ asymptotic region,} \\ v & \rightarrow -m'', \quad y \sim w^{N'_c-\tilde{N}''_c} + \dots \quad \overline{NS5'_R} \text{ asymptotic region,} \\ v & \rightarrow +m'', \quad y \sim w^{N_c+N'_c+N'_f+N'_f+4} + \dots \quad NS5'_L \text{ asymptotic region,} \\ v & \rightarrow +m'', \quad y \sim w^{\tilde{N}''_c-N'_c+2N_c+2N'_f+N'_f+4} + \dots \quad NS5'_R \text{ asymptotic region.} \end{aligned}$$

We denote the mirror branes by writing the bar on the corresponding NS-brane. The two  $NS5'_{L,R}$ -branes are moving in the  $+v$  direction respectively holding everything else fixed instead of moving D6-branes in the  $+v$  direction. The corresponding mirrors of D4-branes are moved appropriately.

## 2.6 Magnetic theories for the multiple product gauge groups

Now one can generalize the method for the triple product gauge groups to the finite  $n$ -multiple product gauge groups characterized by

$$SU(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})$$

with the matter, the  $(n-1)$  bifundamentals  $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_{n-1}, \bar{\square}_n)$ , their complex conjugate  $(n-1)$  fields  $(\bar{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \bar{\square}_{n-1}, \square_n)$ , linking the gauge groups together,  $n$ -fundamentals  $(\square_1, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_n)$ ,  $n$ -antifundamentals  $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \bar{\square}_n)$ , and there exist a symmetric tensor  $(\mathbf{symm}, \mathbf{1}_2, \dots, \mathbf{1}_n)$ , and a conjugate symmetric tensor  $(\overline{\mathbf{symm}}, \mathbf{1}_2, \dots, \mathbf{1}_n)$ . Then the mass-deformed superpotential can be written as  $W_{elec} = \sum_{i=1}^n m_i Q_i \tilde{Q}_i$ . The brane configuration can be constructed from Figure 1 by adding  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes to the right of  $NS5'_R$ -brane (and its mirrors) leading to the fact that any two neighboring NS-branes should be perpendicular to each other.

There exist  $(2n-2)$  magnetic theories and they can be classified as follows.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,1})$

When the Seiberg dual is taken for the first gauge group factor by assuming that  $\Lambda_1 \gg \Lambda_i$  where  $i = 2, \dots, n$ , one follows the procedure given in the subsection 2.2. The gauge group is

$$SU(\tilde{N}_{c,1} \equiv 2N_{f,1} + 2N_{c,2} - N_{c,1}) \times SU(N_{c,2}) \times \dots \times SU(N_{c,n})$$

and the matter contents are given by the dual quarks  $q_1$   $(\square_1, \mathbf{1}, \dots, \mathbf{1}_n)$  and  $\tilde{q}_1$  in the representation  $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1}_n)$  as well as  $(n-1)$  quarks  $Q_i$  and  $\tilde{Q}_i$  where  $i = 2, \dots, n$ , the bifundamentals  $f_1$  in the representation  $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n)$  under the dual gauge group, and  $\tilde{f}_1$  in the representation  $(\bar{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n)$  under the dual gauge group in addition to  $(n-2)$  bifundamentals  $G_i$  and  $\tilde{G}_i$ , a symmetric tensor  $(\mathbf{symm}, \mathbf{1}_2, \dots, \mathbf{1}_n)$ , and a conjugate symmetric tensor  $(\overline{\mathbf{symm}}, \mathbf{1}_2, \dots, \mathbf{1}_n)$ , and various gauge singlets  $X_2, \tilde{X}_2, M_1$  and  $\Phi_2$ . The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 2 (and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_1 q_1 \tilde{s} \tilde{q}_1 + f_1 \tilde{X}_2 \tilde{q}_1 + \tilde{f}_1 q_1 X_2 + \Phi_2 f_1 \tilde{f}_1 \right) + m_1 M_1.$$

By computing the contribution for the one loop as in the subsection 2.2, the vacua are stable and the asymptotic behavior of  $(2n+1)$  NS-branes can be obtained also.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,2})$

When the Seiberg dual is taken for the second gauge group factor by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, \dots, n$ , one follows the procedure given in the subsection 2.3. The gauge group is given by

$$SU(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + N_{c,1} - N_{c,2}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand

side of the  $NS5'_R$ -brane of Figure 3(and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + g_2 \tilde{X}_3 \tilde{q}_2 + \tilde{g}_2 q_2 X_3 + \Phi_3 g_2 \tilde{g}_2 \right) + m_2 M_2.$$

By computing the contribution for the one loop as in the subsection 2.3, the vacua are stable and the asymptotic behavior of  $(2n + 1)$  NS-branes can be obtained.

When the Seiberg dual is taken for the second gauge group factor with different brane motion by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 2.4. The gauge group is given by

$$SU(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + N_{c,1} - N_{c,2}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n - 3)$  NS-branes,  $(n - 3)$  sets of D6-branes and  $(n - 3)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 4(and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + f_1 X_1 q_2 + \tilde{f}_1 \tilde{q}_2 \tilde{X}_1 + \Phi_1 f_1 \tilde{f}_1 \right) + m_2 M_2.$$

By computing the contribution for the one loop as in the subsection 2.4, the vacua are stable and the asymptotic behavior of  $(2n + 1)$  NS-branes can be obtained.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,i})$  where  $3 \leq i \leq n - 1$

When the Seiberg dual is taken for the middle gauge group factor by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i - 1, i + 1, \dots, n$ , one follows the procedure given in the subsection 2.3 of [18]. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i - 2)$  NS-branes,  $(i - 2)$  sets of D6-branes and  $(i - 2)$  sets of D4-branes are present between the  $NS5_M$ -brane and the NS5-brane and the extra  $(n - i - 1)$  NS-branes,  $(n - i - 1)$  sets of D6-branes and  $(n - i - 1)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 3(and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + g_i \tilde{X}_{i+1} \tilde{q}_i + \tilde{g}_i q_i X_{i+1} + \Phi_{i+1} g_i \tilde{g}_i \right) + m_i M_i.$$

By computing the contribution for the one loop as in the subsection 2.3 of [18], the vacua are stable and the asymptotic behavior of  $(2n + 1)$  NS-branes can be obtained.

When the Seiberg dual is taken for the middle gauge group factor with different brane motion by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i-1, i+1, \dots, n$ , one follows the procedure given in the subsection 2.4 of [18]. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i-2)$  NS-branes,  $(i-2)$  sets of D6-branes and  $(i-2)$  sets of D4-branes are present between the  $NS5_M$ -brane and the NS5-brane of Figure 4 and the extra  $(n-i-1)$  NS-branes,  $(n-i-1)$  sets of D6-branes and  $(n-i-1)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 4 (and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + f_{i-1} X_{i-1} q_i + \tilde{f}_{i-1} \tilde{q}_i \tilde{X}_{i-1} + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \right) + m_i M_i.$$

By computing the contribution for the one loop as in the subsection 2.4 of [18], the vacua are stable and the asymptotic behavior of  $(2n+1)$  NS-branes can be obtained.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,n})$

When the Seiberg dual is taken for the last gauge group factor by assuming that  $\Lambda_n \gg \Lambda_i$  where  $i = 1, 2, \dots, (n-1)$ , one follows the procedure given in the subsection 2.5. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present between the  $NS5_M$ -brane and the  $NS5'_L$ -brane of Figure 5 (and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_n q_n \tilde{q}_n + g_{n-1} X_{n-1} q_n + \tilde{g}_{n-1} \tilde{q}_n \tilde{X}_{n-1} + \Phi_{n-1} g_{n-1} \tilde{g}_{n-1} \right) + m_n M_n.$$

By computing the contribution for the one loop as in the subsection 2.5, the vacua are stable and the asymptotic behavior of  $(2n+1)$  NS-branes can be obtained.

### 3 Meta-stable brane configurations of multiple product gauge theories with different matters

#### 3.1 Electric theory

We describe the gauge theory with triple product gauge groups  $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  where the antisymmetric, a conjugate symmetric tensors and eight fundamentals are present in addition to the fundamentals and bifundamentals. The matter contents are



- $N_f$ -chiral multiplets  $Q$  are in the representation  $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$ , and  $N_f$ -chiral multiplets  $\tilde{Q}$  are in the representation  $(\overline{\mathbf{N}}_c, \mathbf{1}, \mathbf{1})$
- $N'_f$ -chiral multiplets  $Q'$  are in the representation  $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ , and  $N'_f$ -chiral multiplets  $\tilde{Q}'$  are in the representation  $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{1})$
- $N''_f$ -chiral multiplets  $Q''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c)$ , and  $N''_f$ -chiral multiplets  $\tilde{Q}''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}}''_c)$
- Eight-chiral multiplets  $\hat{Q}$  are in the representation  $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$
- The flavor-singlet field  $F$  is in the bifundamental representation  $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$ , and its conjugate field  $\tilde{F}$  is in the bifundamental representation  $(\overline{\mathbf{N}}_c, \mathbf{N}'_c, \mathbf{1})$
- The flavor-singlet field  $G$  is in the bifundamental representation  $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}}''_c)$ , and its conjugate field  $\tilde{G}$  is in the bifundamental representation  $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{N}''_c)$
- The flavor-singlet field  $A$ , which is in an antisymmetric tensor representation under the  $SU(N_c)$ , is in the representation  $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c - \mathbf{1}), \mathbf{1}, \mathbf{1})$ , and conjugate field of symmetric tensor field  $\tilde{S}$  is in the representation  $(\frac{1}{2}\mathbf{N}_c(\mathbf{N}_c + \mathbf{1}), \mathbf{1}, \mathbf{1})$

In this case also if we ignore the antisymmetric and conjugate symmetric tensors  $A$  and  $\tilde{S}$  and eight-fundamentals  $\hat{Q}$ , this theory was studied in [13, 18]. If we put to  $Q'', \tilde{Q}'', G, \tilde{G}, A, \hat{Q}$  and  $\tilde{S}$  zero, then this becomes the product gauge group theory with fundamentals and bifundamentals [4, 14, 13, 19]. Furthermore, if we put to  $Q', \tilde{Q}', Q'', \tilde{Q}'', F, \tilde{F}, G$ , and  $\tilde{G}$  zero, then this becomes the single gauge group theory with fundamentals and bifundamentals, eight-fundamentals, antisymmetric and conjugate symmetric tensors [8, 9, 10].

The coefficient of the beta function of the first gauge group is given by  $b_{SU(N_c)} = 3N_c - (N_f + 4) - N'_c - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2)$  by realizing the index of the antisymmetric and symmetric representations of  $SU(N_c)$  gauge group and the coefficient of the beta function of the second gauge group is given by  $b_{SU(N'_c)} = 3N'_c - N'_f - N_c - N''_c$  and finally the coefficient of the beta function of the third gauge group is given by  $b_{SU(N''_c)} = 3N''_c - N''_f - N'_c$ . We'll see how these coefficients change in the magnetic theory. We denote the strong coupling scales for  $SU(N_c)$  as  $\Lambda_1$ , for  $SU(N'_c)$  as  $\Lambda_2$  and for  $SU(N''_c)$  as  $\Lambda_3$  respectively.

The electric superpotential with mass-deformed terms and an interaction term between eight-fundamentals and conjugate symmetric tensor field is

$$\begin{aligned}
W_{elec} = & \left( \mu A_d^2 + \lambda Q A_d \tilde{Q} + A A_d \tilde{S} + \tilde{F} A_d F + \mu' A_d'^2 + \lambda' Q' A_d' \tilde{Q}' + \tilde{F} A_d' F + \tilde{G} A_d' G \right. \\
& \left. + \mu'' A''^2 + \lambda'' Q'' A'' \tilde{Q}'' + \tilde{G} A'' G \right) + \hat{Q} \tilde{S} \hat{Q} + m Q \tilde{Q} + m' Q' \tilde{Q}' + m'' Q'' \tilde{Q}'' .
\end{aligned}$$

After integrating the adjoint fields  $A_d$  for  $SU(N_c)$ ,  $A'_d$  for  $SU(N'_c)$  and  $A''_d$  for  $SU(N''_c)$  and taking  $\mu, \mu'$  and  $\mu''$  to infinity limit which is equivalent to take any two NS-branes

be perpendicular to each other, the mass-deformed electric superpotential becomes  $W_{elec} = \hat{Q}\tilde{S}\hat{Q} + mQ\tilde{Q} + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''$ .

The type IIA brane configuration for this mass-deformed theory can be described by as follows. The  $N_c$ -color D4-branes (01236) are suspended between the  $NS5'_M$ -brane (012389) located at  $x^6 = 0$  and the  $NS5_L$ -brane (012345) along positive  $x^6$  direction, together with  $N_f$  D6-branes (0123789) which are parallel to  $NS5'_M$ -brane and have nonzero  $v$  direction. The  $NS5'$ -brane is located at the right hand side of the  $NS5_L$ -brane along the positive  $x^6$  direction and there exist  $N'_c$ -color D4-branes suspended between them, with  $N'_f$  D6-branes which have nonzero  $v$  direction. Moreover, the  $NS5_R$ -brane is located at the right hand side of the  $NS5'$ -brane along the positive  $x^6$  direction and there exist  $N''_c$ -color D4-branes suspended between them, with  $N''_f$  D6-branes which have nonzero  $v$  direction. There exist two types of orientifold 6-plane (0123789) at the origin  $x^6 = 0$  and they act as  $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ . Then the mirrors of above branes appear in the negative  $x^6$  region and are denoted by bar on the corresponding branes. From the left to the right, there are  $\overline{NS5_R}$ -,  $\overline{NS5'}$ -,  $\overline{NS5_L}$ -,  $NS5'_M$ -,  $NS5_L$ -,  $NS5'$ -, and  $NS5_R$ -branes.

We summarize the  $\mathcal{N} = 1$  supersymmetric electric brane configuration in type IIA string theory as follows:

- Four NS5-branes in (012345) directions.
- Three NS5'-branes in (012389) directions.
- Two sets of  $N_c(N'_c)[N''_c]$ -color D4-branes in (01236) directions.
- Two sets of  $N_f(N'_f)[N''_f]$  D6-branes in (0123789) directions.
- Eight half D6-branes in (0123789) directions.
- Two  $O6^\pm$ -planes in (0123789) directions with  $x^6 = 0$

Now we draw this electric brane configuration in Figure 6 and we put the coincident  $N_f(N'_f)[N''_f]$  D6-branes with positive  $x^6$  in the nonzero  $v$  direction in general. This brane configuration can be obtained from the brane configuration of [15] by adding the two outer NS5-branes(i.e.,  $\overline{NS5_R}$ -brane and  $NS5_R$ -brane), two sets of  $N''_c$  D4-branes and two sets of  $N''_f$  D6-branes or from the one of [13] with the gauge theory of triple product gauge groups by adding O6-planes and the extra NS-branes, D4-branes and D6-branes. Then the mirrors with negative  $x^6$  can be constructed by using the action of O6-plane and are located at the positions by changing (456) directions for original branes with minus signs.

### 3.2 Magnetic theory with dual for first gauge group

By moving the mirror of  $NS5_L$ -brane to the right all the way past O6-plane(and the  $NS5_L$ -brane to the left) we arrive at the Figure 7A. Then the linking number of  $NS5_L$ -brane from

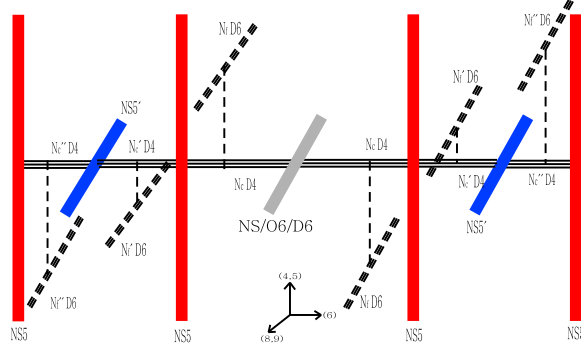


Figure 6: The  $\mathcal{N} = 1$  supersymmetric electric brane configuration with  $SU(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(Q')[Q'']$  and  $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(G), \tilde{F}(\tilde{G})$ , an antisymmetric tensor  $A$ , a conjugate symmetric tensor  $\tilde{S}$  and eight-fundamentals. The  $O6^\pm$ -planes are located at the origin  $x^6 = 0$ . The two NS5-branes with positive  $x^6$  can be denoted by  $NS5_{L,R}$ -branes. At the origin of  $(x^6, v, w)$  coordinates, there exist NS5'-brane,  $O6^+$ -plane,  $O6^-$ -plane and eight half-D6-branes. One denotes this combination as  $NS5/O6/D6$ -branes here.

Figure 7A becomes  $L_5 = \frac{N_f}{2} + \frac{1}{2}(4) - \tilde{N}_c + N'_c + N_f$ . Originally, the linking number was  $L_5 = -\frac{N_f}{2} - \frac{1}{2}(4) + N_c - N'_c$  from Figure 6. This implies that the number of D4-branes in magnetic theory,  $\tilde{N}_c$ , becomes [11]

$$\tilde{N}_c = 2(N_f + N'_c) - N_c + 4.$$

Let us draw this magnetic brane configuration in Figure 7A and recall that we put the coincident  $N_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q'$  and  $Q''$  by putting  $N'_f$  and  $N''_f$  D6-branes at  $v = 0$ . Because  $N'_c$  or  $N''_c$  D4-branes are suspending between two equal  $NS5_{L,R}$ -branes located at different  $x^6$  coordinate, these D4-branes can slide along the  $v$ -direction. If we ignore the NS5'-brane,  $N'_c$  D4-branes,  $N'_f$  D6-branes, the  $NS5_R$ -brane,  $N''_c$  D4-branes and  $N''_f$  D6-branes (detaching these branes from Figure 7A), then this brane configuration leads to the  $\mathcal{N} = 1$  magnetic theory with gauge group  $SU(\tilde{N}_c = 2N_f - N_c + 4)$  with  $N_f$  massive fundamental flavors plus antisymmetric, conjugate symmetric flavors, eight-fundamentals and gauge singlets [21, 11]. On the other hand, when we ignore the  $NS5_R$ -brane,  $N'_c$  D4-branes and  $N'_f$  D6-branes (detaching these branes from Figure 7A), then this brane configuration leads to the  $\mathcal{N} = 1$  magnetic theory with gauge group  $SU(\tilde{N}_c = 2N_f + 2N'_c - N_c + 4) \times SU(N'_c)$  with fundamental flavors, bifundamentals, antisymmetric, conjugate symmetric flavors, eight-fundamentals and gauge singlets in Figure 6 of [15].

Now let us recombine  $\tilde{N}_c$  flavor D4-branes among  $N_f$  flavor D4-branes (connecting between D6-branes and  $NS5_L$ -brane) with the same number of color D4-branes (connecting between

$NS5'_M$ -brane and  $NS5_L$ -brane) and push them in  $+v$  direction from Figure 7A. We assume that  $N_c \geq N_f + 2N'_c + 4$ . After this procedure, there are no color D4-branes between  $NS5'_M$ -brane and  $NS5_L$ -brane. For the flavor D4-branes, we are left with only  $(N_f - \tilde{N}_c) = N_c - N_f - 2N'_c - 4$  flavor D4-branes connecting between D6-branes and  $NS5'_M$ -brane.

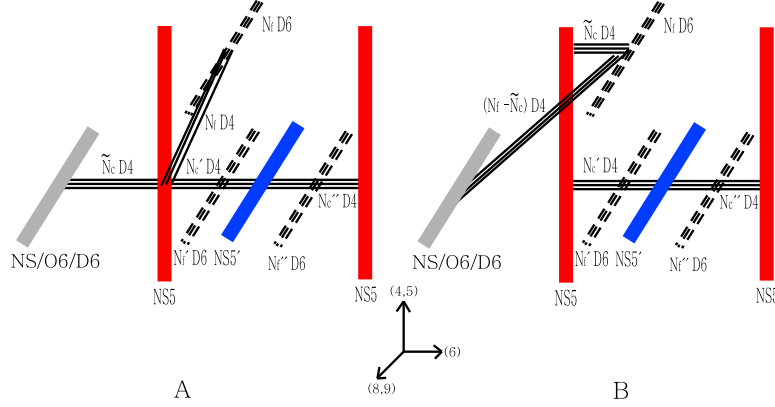


Figure 7: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(\tilde{N}_c = 2N_f + 2N'_c - N_c + 4) \times SU(N'_c) \times SU(N'_c)$  gauge group with fundamentals  $q(Q')[Q'']$  and  $\tilde{q}(\tilde{Q}')[\tilde{Q}'']$  for each gauge group,  $\hat{q}$ , and bifundamentals  $f(G)$  and  $\tilde{f}(\tilde{G})$ ,  $a$  and  $\tilde{s}$ , and gauge singlets in Figure 7A. In Figure 7B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q'(Q'')$  and  $\tilde{Q}'(\tilde{Q}'')$  is given.

The additional  $N_f$ - $SU(N'_c)$  fundamentals  $X'$  and  $N_f$ - $SU(N'_c)$  antifundamentals  $\tilde{X}'$  are originating from the  $SU(N_c)$  chiral mesons  $\tilde{F}Q$  and  $F\tilde{Q}$  respectively. Therefore, there are free indices for a single color and a single flavor. Then the strings stretching between the  $N_f$  D6-branes and  $N'_c$  D4-branes will give rise to these additional  $N_f$ - $SU(N'_c)$  fundamentals and  $N_f$ - $SU(N'_c)$  antifundamentals. The gauge singlet  $M$  corresponds to the  $SU(N_c)$  chiral meson  $Q\tilde{Q}$  where the color indices are contracted. Because the  $N_f$  D6-branes are parallel to the  $NS5'_M$ -brane from Figure 7B, the newly created  $N_f$ -flavor D4-branes can slide along the plane consisting of these  $N_f$  D6-branes and  $NS5'_M$ -brane freely. The fluctuations of the gauge-singlet  $M$  correspond to the motion of  $N_f$  flavor D4-branes along (789) directions in Figure 7B. For the nonsupersymmetric brane configuration, a misalignment for the  $N_f$ -flavor D4-branes arises and some of the vacuum expectation value of  $M$  is fixed and the remaining components are arbitrary. The  $\Phi'$  corresponds to the  $SU(N_c)$  chiral meson  $F\tilde{F}$  where the color indices for the first gauge group are contracted each other. The fluctuations of the singlet  $\Phi'$  correspond to the motion of  $N'_c$  D4-branes suspended two  $NS5_{L,R}$ -branes along the  $v$  direction in Figure 7B. Although the gauge singlets  $N, \hat{M}, P$  and  $\tilde{P}$  appear in the dual magnetic superpotential for the general rotation angles of NS-branes and D6-branes, the case we are considering does not contain these gauge singlets, as observed in [11].

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$
matter :	$q_f \oplus \tilde{q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$\hat{q}_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 8)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$a \oplus \tilde{s}$	$(\text{asymm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\text{symm}}, \mathbf{1}, \mathbf{1})$
	$(X'_n \equiv) \tilde{F}Q \oplus F\tilde{Q} (\equiv \tilde{X}'_{\tilde{n}})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n, \tilde{n} = 1, \dots, N_f)$
	$(M_{f,\tilde{g}} \equiv) Q\tilde{Q}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{g} = 1, \dots, N_f)$
	$(\hat{M}_{f,\tilde{g}} \equiv) \hat{Q}\tilde{Q}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 8, \tilde{g} = 1, \dots, N_f)$
	$(\Phi' \equiv) F\tilde{F}$	$(\mathbf{1}, \text{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$
	$(P_{f,g} \equiv) Q\tilde{S}Q \oplus \tilde{Q}A\tilde{Q} (\equiv \tilde{P}_{\tilde{f},\tilde{g}})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f}, g, \tilde{g} = 1, \dots, N_f)$
	$(N_{f,\tilde{g}} \equiv) Q\tilde{S}A\tilde{Q}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{g} = 1, \dots, N_f)$

The coefficient of the beta function of the first dual gauge group factor is given by  $b_{SU(\tilde{N}_c)}^{mag} = 3\tilde{N}_c - (N_f + 4) - N'_c - \frac{1}{2}(\tilde{N}_c + 2) - \frac{1}{2}(\tilde{N}_c - 2)$  and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - \tilde{N}_c - N''_c - N_f - N'_c$  and the coefficient of the beta function of the third gauge group factor is given by  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - N'_c = b_{SU(N''_c)}$ .

From the superpotential obtained from [8, 9, 10] partly

$$W_{dual} = \left( Mq\tilde{s}a\tilde{q} + mM + \hat{q}\tilde{s}\hat{q} + \hat{M}\hat{q}\hat{q} \right) + \tilde{f}X'q + f\tilde{q}\tilde{X}' + \Phi'f\tilde{f},$$

then,  $q\tilde{s}a\tilde{q}$  has rank  $\tilde{N}_c$  while  $m$  has a rank  $N_f$ . Therefore, the derivative of the superpotential  $W_{dual}$  with respect to  $M$ , cannot be satisfied if the rank  $N_f$  exceeds  $\tilde{N}_c$  and the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $a\tilde{q}M = 0 = Mq\tilde{s}, q\tilde{s}a\tilde{q} + m = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q\tilde{s} \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \langle a\tilde{q} \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = \langle \hat{q} \rangle = \langle \hat{M} \rangle = 0. \end{aligned}$$

Let us expand around on a point on the vacua, as done in [1]. Then the remaining relevant terms of superpotential are given by  $W_{dual}^{rel} = M_0 \left( \delta\hat{\varphi} \delta\hat{\tilde{\varphi}} + m \right) + \delta Z \delta\hat{\varphi} a_0 \tilde{q}_0 + \delta\tilde{Z} q_0 \tilde{s}_0 \delta\hat{\tilde{\varphi}}$

by following the fluctuations for the various fields in [11]. Note that there exist five kinds of terms, the vacuum  $\langle q \rangle$  multiplied by  $\delta\tilde{f}\delta X'$ , the vacuum  $\langle \tilde{q} \rangle$  multiplied by  $\delta\tilde{X}'\delta f$ , the vacuum  $\langle \Phi' \rangle$  multiplied by  $\delta f\delta\tilde{f}$ , the vacuum  $\langle \tilde{s} \rangle$  multiplied by  $\delta\hat{q}\delta\hat{q}$ , and the vacuum  $\tilde{q}$  multiplied by  $\delta\hat{M}\delta\hat{q}$ . By redefining these as before, they do not enter the contributions for the one loop result, up to quadratic order. As done in [7], the defining function  $\mathcal{F}(v^2)$  can be computed and using the equation (2.14) of [22] of  $m_{M_0}^2$  and  $\mathcal{F}(v^2)$ , one gets that  $m_{M_0}^2$  will contain  $(\log 4 - 1) > 0$  implying that these are stable.

The nonsupersymmetric minimal energy brane configuration Figure 7B with a replacement  $N_f''$  D6-branes by the NS5'-brane(neglecting the  $NS5_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes and  $N_f'$  D6-branes) leads to the Figure 7B of [23] with a rotation of NS5-brane by  $\frac{\pi}{2}$  angle.

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Now the asymptotic regions of various NS-branes can be determined by reading off the first two terms of the seventh order curve above giving the  $\overline{NS5_R}$ -brane asymptotic region, next two terms giving the  $\overline{NS5'}$ -brane asymptotic region, next two terms giving the  $\overline{NS5_L}$ -brane asymptotic region, next two terms giving  $NS5'_M$ -brane asymptotic region, next two terms giving  $NS5_L$ -brane asymptotic region, next two terms giving NS5'-brane asymptotic region, and final two terms giving  $NS5_R$ -brane asymptotic region. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{N_c''} + \dots & \overline{NS5_R} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{-\tilde{N}_c + N_c' + N_f'' + N_f + N_f' + 4} + \dots & NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{\tilde{N}_c - N_c' + N_f'' + N_f + N_f'} + \dots & \overline{NS5_L} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{-N_c'' + 2N_f'' + 2N_f + 2N_f' + 4} + \dots & NS5_R \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow -m, & y &\sim w^{N_c' + N_f'' - N_c''} + \dots & \overline{NS5'} \text{ asymptotic region,} \\ v &\rightarrow +m, & y &\sim w^{N_f'' + N_f + N_f' + 2} + \dots & NS5'_M \text{ asymptotic region,} \\ v &\rightarrow +m, & y &\sim w^{N_f'' + 2N_f + 2N_f' - N_c' + N_c'' + 4} + \dots & NS5' \text{ asymptotic region.} \end{aligned}$$

### 3.3 Magnetic theory with dual for second gauge group

By moving the NS5'-brane in Figure 6 with massive  $N_f'$  D6-branes to the left all the way past the  $NS5_L$ -brane, one arrives at the Figure 8A. The linking number of NS5'-brane from

Figure 8A is  $L_5 = -\frac{N'_f}{2} + \tilde{N}'_c - N_c$  while the linking number of NS5'-brane from Figure 6 is  $L_5 = \frac{N'_f}{2} + N''_c - N'_c$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N''_c + N_c - N'_c.$$

Let us draw this magnetic brane configuration in Figure 8A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore the mirror branes corresponding to  $NS5'_{L,R}$ -branes, NS5-brane,  $N_f(N'_f)[N''_f]$  D6-branes and  $N_c(N'_c)[N''_c]$  D4-branes, O6-planes and half D6-branes(detaching these branes from Figure 8A), then this brane configuration looks similar to the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  with fundamentals, bifundamentals, and singlets in Figure 4 of [18].

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and  $NS5_L$ -brane) with the same number of color D4-branes(connecting between NS5'-brane and  $NS5_L$ -brane) and push them in  $+v$  direction from Figure 8A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and NS5'-brane.

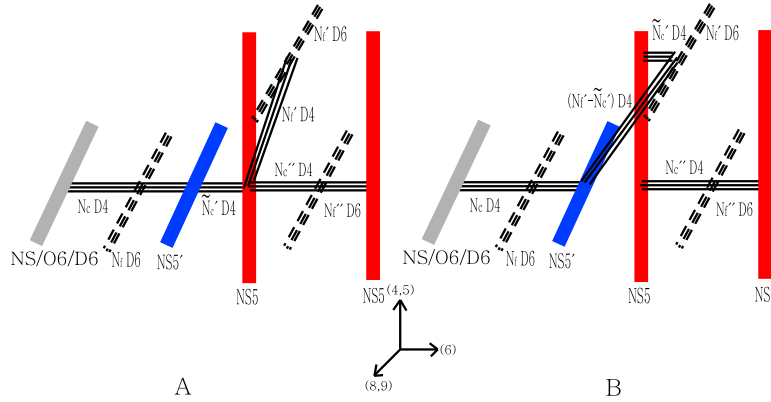


Figure 8: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q)[Q'']$  and  $\tilde{Q}(\tilde{q})[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(g)$ ,  $\hat{Q}$ ,  $\tilde{F}(\tilde{g})$ ,  $A$  and  $\tilde{S}$ , and gauge singlets in Figure 8A. In Figure 8B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $\tilde{Q}(\tilde{Q}'')$  is given.

The additional  $N'_f$ - $SU(N''_c)$  fundamentals  $X''$  and  $N'_f$ - $SU(N''_c)$  antifundamentals  $\tilde{X}''$  are originating from the  $SU(N'_c)$  chiral mesons  $\tilde{G}Q'$  and  $G\tilde{Q}'$  respectively. The gauge singlet  $M'$

corresponds to the  $SU(N'_c)$  chiral meson  $Q'\tilde{Q}'$  where the color indices are contracted. The fluctuations of the gauge-singlet  $M'$  correspond to the motion of  $N'_f$  flavor D4-branes along (789) directions in Figure 8B. The  $\Phi''$  corresponds to the  $SU(N'_c)$  chiral meson  $G\tilde{G}$  where the color indices for the second gauge group are contracted each other.

The coefficient of the beta function of the first gauge group factor is given by  $b_{SU(N_c)}^{mag} = 3N_c - (N_f + 4) - \tilde{N}'_c - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2)$  as before and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$ . The coefficient of the beta function of the third gauge group factor is given by  $b_{SU(N''_c)}^{mag} = 3N''_c - N'_f - \tilde{N}'_c - N'_f - N''_c$ .

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$\hat{Q}_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 8)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$A \oplus \tilde{S}$	$(\mathbf{asymm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{symm}}, \mathbf{1}, \mathbf{1})$
	$(X''_{n'} \equiv) \tilde{G}Q' \oplus G\tilde{Q}' (\equiv \tilde{X}''_{\tilde{n}'})$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi'' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

From the superpotential

$$W_{dual} = (M'q'\tilde{q}' + m'M') + X''\tilde{g}q' + \tilde{X}''\tilde{q}'g + \Phi''g\tilde{g}$$

one sees that  $q'\tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $\tilde{q}'M' = 0 = M'q', q'\tilde{q}' + m' = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'}e^{\phi}\mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'}e^{-\phi}\mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X'' \rangle = \langle \tilde{X}'' \rangle = 0. \end{aligned}$$

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.



The Figure 8B with a replacement  $N'_f$  D6-branes by the NS5'-brane(neglecting the  $NS5_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes and  $N_f$  D6-branes) leads to the Figure 8B of [23].

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{N''_c} + \dots & \overline{NS5_R} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{-\tilde{N}'_c + 3N_c + N''_c + N''_f + N'_f + 6} + \dots & NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{\tilde{N}'_c - N''_c + N''_f + N'_f} + \dots & \overline{NS5_L} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{-N''_c + 2N_c + N''_f + 2N'_f + 4} + \dots & NS5_R \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow -m', & y &\sim w^{N_c + N''_f - \tilde{N}'_c + N'_f} + \dots & \overline{NS5'} \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{N_c + N''_f + N'_f + 2} + \dots & NS5'_M \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{N''_f + N'_f + \tilde{N}'_c + 2} + \dots & NS5' \text{ asymptotic region.} \end{aligned}$$

### 3.4 Magnetic theory with dual for second gauge group

By moving the  $NS5_L$ -brane in Figure 6 with massive  $N'_f$  D6-branes to the right all the way past the NS5'-brane, one arrives at the Figure 9A. The linking number of  $NS5_L$ -brane from Figure 9A is  $L_5 = \frac{N'_f}{2} - \tilde{N}'_c + N''_c$  and the linking number of  $NS5_L$ -brane from the Figure 6 is  $L_5 = -\frac{N'_f}{2} + N'_c - N_c$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N''_c + N_c - N'_c.$$

Let us draw this magnetic brane configuration in Figure 9A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore  $NS5_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes(detaching these branes from Figure 9A), then this brane configuration leads to the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N_c - N'_c)$  with fundamentals, bifundamentals, eight-fundamentals, antisymmetric, a conjugate symmetric flavors, and singlets in Figure 7 of [15].

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and NS5'-brane) with the same number of color D4-branes(connecting between

NS5'-brane and  $NS5_L$ -brane) and push them in  $+v$  direction from Figure 9A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and NS5'-brane.

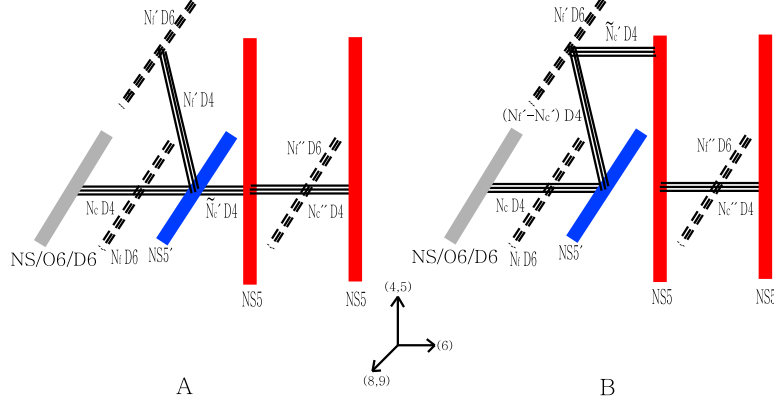


Figure 9: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(\tilde{N}'_c = N'_f + N''_c + N_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q)[Q'']$  and  $\tilde{Q}(\tilde{q})[\tilde{Q}'']$  for each gauge group and bifundamentals  $f(G)$  and  $\tilde{f}(\tilde{G})$ ,  $A$  and  $\tilde{S}$ ,  $\tilde{Q}$ , and gauge singlets in Figure 9A. In Figure 9B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $\tilde{Q}(\tilde{Q}'')$  is given.

The additional  $N'_f$ - $SU(N_c)$  fundamentals  $X$  and  $N'_f$ - $SU(N_c)$  antifundamentals  $\tilde{X}$  are originating from the  $SU(N'_c)$  chiral mesons  $FQ'$  and  $\tilde{F}\tilde{Q}'$  respectively. The gauge singlet  $M'$  corresponds to the  $SU(N'_c)$  chiral meson  $Q'\tilde{Q}'$  where the color indices are contracted. The fluctuations of the gauge-singlet  $M'$  correspond to the motion of  $N'_f$  flavor D4-branes along (789) directions in Figure 9B. The  $\Phi$  corresponds to the  $SU(N'_c)$  chiral meson  $F\tilde{F}$  where the color indices for the second gauge group are contracted each other.

The coefficient of the beta function of the first gauge group factor is given by  $b_{SU(N_c)}^{mag} = 3N_c - (N_f + 4) - \tilde{N}'_c - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2) - N'_f - N_c$  and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$ . The coefficient of the beta function of the third gauge group factor is given by  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c$ .

From the superpotential

$$W_{dual} = (M'q\tilde{q} + m'M') + Xfq' + \tilde{X}\tilde{q}\tilde{f} + \Phi f\tilde{f}$$

one sees that  $q\tilde{q}$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $\tilde{q}'M' = 0 = M'q', q\tilde{q} + m' = 0$ . Then the solutions

can be written as

$$\begin{aligned}
\langle q' \rangle &= \begin{pmatrix} \sqrt{m'} e^\phi \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'} e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\
\langle f \rangle &= \langle \tilde{f} \rangle = \langle X \rangle = \langle \tilde{X} \rangle = 0.
\end{aligned}$$

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$\hat{Q}_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 8)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$A \oplus \tilde{S}$	$(\mathbf{asymm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{symm}}, \mathbf{1}, \mathbf{1})$
	$(X_{n'} \equiv) FQ' \oplus \tilde{F}\tilde{Q}' (\equiv \tilde{X}_{\tilde{n}'})$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi \equiv) F\tilde{F}$	$(\mathbf{adj}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The nonsupersymmetric minimal energy brane configuration Figure 9B (neglecting the O6-planes, half D6-branes, and the mirrors) looks similar to the Figure 3B of [18] and leads to a reflection of Figure 3B of [18] with respect to NS5-brane.

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned}
w &\rightarrow 0, \quad y \sim v^{N''_c} + \dots \quad \overline{NS5}_R \text{ asymptotic region,} \\
w &\rightarrow 0, \quad y \sim v^{-\tilde{N}'_c + N''_c + N'_f + N_f + 2N'_f + 4} + \dots \quad NS5_L \text{ asymptotic region,} \\
w &\rightarrow 0, \quad y \sim v^{\tilde{N}'_c - N''_c + N'_f} + \dots \quad \overline{NS5}_L \text{ asymptotic region,} \\
w &\rightarrow 0, \quad y \sim v^{-N''_c + 2N'_f + N_f + 2N'_f + 6} + \dots \quad NS5_R \text{ asymptotic region.}
\end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow -m', & y &\sim w^{N_c+N_f''+2-\tilde{N}_c'} + \dots & \overline{NS5'} \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{N_f''+N_f+N_f'+4} + \dots & NS5'_M \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{N_f''+N_f+2N_f'-\tilde{N}_c'-N_c+4} + \dots & NS5' \text{ asymptotic region.} \end{aligned}$$

### 3.5 Magnetic theory with dual for third gauge group

By moving the  $NS5_R$ -brane to the left all the way past the  $NS5'$ -brane, one arrives at the Figure 10A. The linking number of  $NS5_R$ -brane from Figure 10A is given by  $L_5 = \frac{N_f''}{2} - \tilde{N}_c''$  and the linking number of  $NS5_R$ -brane from Figure 6 is  $L_5 = -\frac{N_f''}{2} + N_c'' - N_c'$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c'' = N_f'' + N_c' - N_c''.$$

Let us draw this magnetic brane configuration in Figure 10A and recall that we put the coincident  $N_f''$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q'$  by putting  $N_f$  and  $N_f'$  D6-branes at  $v = 0$ . If we ignore the mirror branes corresponding to  $NS5'_{L,R}$ -branes,  $NS5$ -brane,  $N_f(N_f')[N_f'']$  D6-branes and  $N_c(N_c')[N_c'']$  D4-branes, O6-planes and half-D6-branes(detaching these branes from Figure 10A), then this brane configuration looks similar to the standard  $\mathcal{N} = 1$  magnetic gauge theory with  $SU(N_c) \times SU(N_c') \times SU(\tilde{N}_c'' = N_f'' + N_c' - N_c'')$  with fundamentals, bifundamentals, and singlets in Figure 5 of [18].

Now let us recombine  $\tilde{N}_c''$  flavor D4-branes among  $N_f''$  flavor D4-branes(connecting between D6-branes and  $NS5'$ -brane) with the same number of color D4-branes(connecting between  $NS5_R$ -brane and  $NS5'$ -brane) and push them in  $+v$  direction from Figure 10A. For the flavor D4-branes, we are left with only  $(N_f'' - \tilde{N}_c'') = N_c'' - N_c'$  flavor D4-branes connecting between D6-branes and  $NS5'$ -brane.

The additional  $N_f''$ - $SU(N_c')$  fundamentals  $X'$  and  $N_f''$ - $SU(N_c')$  antifundamentals  $\tilde{X}'$  are originating from the  $SU(N_c'')$  chiral mesons  $GQ''$  and  $\tilde{G}\tilde{Q}''$  respectively. The gauge singlet  $M''$  corresponds to the  $SU(N_c'')$  chiral meson  $Q''\tilde{Q}''$  where the color indices are contracted. The fluctuations of the gauge-singlet  $M''$  correspond to the motion of  $N_f''$  flavor D4-branes along (789) directions in Figure 10A. The  $\Phi'$  corresponds to the  $SU(N_c'')$  chiral meson  $G\tilde{G}$  where the color indices for the third gauge group are contracted each other.

The coefficient of the beta function of the first gauge group factor is given by  $b_{SU(N_c)}^{mag} = 3N_c - (N_f + 4) - N_c' - \frac{1}{2}(N_c + 2) - \frac{1}{2}(N_c - 2) = b_{SU(N_c)}$  and the coefficient of the beta function of the second gauge group factor is given by  $b_{SU(N_c')}^{mag} = 3N_c' - N_f' - N_c - \tilde{N}_c'' - N_f'' - N_c'$ . The coefficient

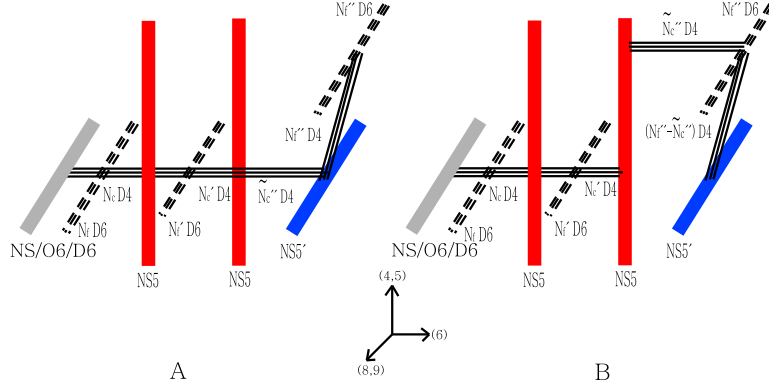


Figure 10: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N_f'' + N'_c - N_c'')$  gauge group with fundamentals  $Q(Q')[q'']$  and  $\tilde{Q}(\tilde{Q}')[\tilde{q}']$  for each gauge group, bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ ,  $A$  and  $\tilde{S}$ ,  $\hat{Q}$ , and gauge singlets in Figure 10A. In Figure 10B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q')$  and  $\tilde{Q}(\tilde{Q}')$  is given.

of the beta function of the third gauge group factor is given by  $b_{SU(\tilde{N}_c'')}^{mag} = 3\tilde{N}_c'' - N_f'' - N'_c$ . Since  $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} > 0$ ,  $SU(N'_c)$  is more asymptotically free than  $SU(N'_c)^{mag}$ .

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$q''_{f''} \oplus \tilde{q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N_f'')$
	$\hat{Q}_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 8)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$A \oplus \tilde{S}$	$(\text{asymm}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\text{symm}}, \mathbf{1}, \mathbf{1})$
	$(X'_{n''} \equiv) GQ'' \oplus \tilde{G}\tilde{Q}'' (\equiv \tilde{X}'_{\tilde{n}''})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n'', \tilde{n}'' = 1, \dots, N_f'')$
	$(M''_{f'', \tilde{g}''} \equiv) Q''\tilde{Q}''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f'', \tilde{g}'' = 1, \dots, N_f'')$
	$(\Phi' \equiv) G\tilde{G}$	$(\mathbf{1}, \text{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The superpotential is

$$W_{dual} = (M'' q'' \tilde{q}'' + m'' M'') + X' g q'' + \tilde{X}' \tilde{q}'' \tilde{g} + \Phi' g \tilde{g}.$$

Then,  $q'' \tilde{q}''$  has rank  $\tilde{N}_c''$  while  $m''$  has a rank  $N_f''$ . The derivative of the superpotential  $W_{dual}$  with respect to  $M''$ , cannot be satisfied if the rank  $N_f''$  exceeds  $\tilde{N}_c''$  and the supersymmetry is

broken. The classical moduli space of vacua can be obtained from F-term equations. Then, it is easy to see that  $\tilde{q}'' M'' = 0 = M'' q'', q'' \tilde{q}'' + m'' = 0$ . Then the solutions can be written as

$$\begin{aligned} \langle q'' \rangle &= \begin{pmatrix} \sqrt{m''} e^\phi \mathbf{1}_{\tilde{N}_c''} \\ 0 \end{pmatrix}, \langle \tilde{q}'' \rangle = \begin{pmatrix} \sqrt{m''} e^{-\phi} \mathbf{1}_{\tilde{N}_c''} & 0 \end{pmatrix}, \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0'' \mathbf{1}_{N_f'' - \tilde{N}_c''} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

One can check that states are stable by realizing the mass of  $m_{M_0''}^2$  positive, by expanding the fields around the vacua.

The nonsupersymmetric minimal energy brane configuration Figure 10B (neglecting the O6-plane, half D6-branes, and the mirrors) looks similar to the Figure 5B of [18] and leads to a reflection of Figure 5B of [18] with respect to NS5-brane.

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, \quad y \sim v^{N_c' - \tilde{N}_c''} + \dots \quad \overline{NS5_R} \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{N_c + N_c' + N_f'' + N_f' + 4} + \dots \quad NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{N_c - N_c' + N_f'' + N_f'} + \dots \quad \overline{NS5_L} \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{\tilde{N}_c'' - N_c' + 2N_c + 2N_f'' + N_f' + 4} + \dots \quad NS5_R \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow -m'', \quad y \sim w^{\tilde{N}_c''} + \dots \quad \overline{NS5'} \text{ asymptotic region,} \\ v &\rightarrow +m'', \quad y \sim w^{N_c + N_f'' + N_f' + 2} + \dots \quad NS5'_M \text{ asymptotic region,} \\ v &\rightarrow +m'', \quad y \sim w^{2N_f'' + N_f' + 2N_c - \tilde{N}_c'' + 4} + \dots \quad NS5' \text{ asymptotic region.} \end{aligned}$$

### 3.6 Magnetic theories for the multiple product gauge groups

Now one can generalize the method for the triple product gauge groups to the finite  $n$ -multiple product gauge groups characterized by

$$SU(N_{c,1}) \times SU(N_{c,2}) \times \dots \times SU(N_{c,n})$$

with the matter, the  $(n-1)$  bifundamentals  $(\square_1, \overline{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_{n-1}, \overline{\square}_n)$ , their complex conjugate  $(n-1)$  fields  $(\overline{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \overline{\square}_{n-1}, \square_n)$ , linking the gauge groups together,  $n$ -fundamentals  $(\square_1, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_n)$ , and

$n$ -antifundamentals  $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1}_n), \dots, (\mathbf{1}_1, \dots, \mathbf{1}, \bar{\square}_n)$ , eight-fundamentals, an antisymmetric tensor, and a conjugate symmetric tensor. Then the mass-deformed superpotential can be written as  $W_{elec} = \sum_{i=1}^n m_i Q_i \tilde{Q}_i$ . The brane configuration can be constructed from Figure 6 by adding  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes to the right of  $NS5_R$ -brane (and its mirrors) leading to the fact that any two neighboring NS-branes should be perpendicular to each other.

There exist  $(2n-2)$  magnetic theories and they can be classified as follows.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,1})$

When the Seiberg dual is taken for the first gauge group factor by assuming that  $\Lambda_1 \gg \Lambda_i$  where  $i = 2, \dots, n$ , one follows the procedure given in the subsection 3.2. The gauge group is

$$SU(\tilde{N}_{c,1} \equiv 2N_{f,1} + 2N_{c,2} - N_{c,1} + 4) \times SU(N_{c,2}) \times \dots \times SU(N_{c,n})$$

and the matter contents are given by the dual quarks  $q_1$  ( $\square_1, \mathbf{1}, \dots, \mathbf{1}_n$ ) and  $\tilde{q}_1$  in the representation  $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1}_n)$  as well as  $(n-1)$  quarks  $Q_i$  and  $\tilde{Q}_i$  where  $i = 2, \dots, n$ , the bifundamentals  $f_1$  in the representation  $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n)$  under the dual gauge group, and  $\tilde{f}_1$  in the representation  $(\bar{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n)$  under the dual gauge group in addition to  $(n-2)$  bifundamentals  $G_i$  and  $\tilde{G}_i$ , eight-fundamentals, an antisymmetric tensor, a conjugate symmetric tensor, and various gauge singlets  $X_2, \tilde{X}_2, M_1, \hat{M}$  and  $\Phi_2$  in addition to  $P, \tilde{P}$  and  $N$ . The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 7 (and their mirrors). The magnetic superpotential can be written as

$$W_{dual} = \left( M_1 q_1 \tilde{s} a \tilde{q}_1 + \hat{q} \tilde{s} \hat{q} + \hat{M} \hat{q} \tilde{q} + f_1 \tilde{X}_2 \tilde{q}_1 + \tilde{f}_1 q_1 X_2 + \Phi_2 f_1 \tilde{f}_1 \right) + m_1 M_1.$$

By computing the contribution for the one loop as in the subsection 3.2, the vacua are stable and the asymptotic behavior of  $(2n+1)$  NS-branes can be obtained also.

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,2})$

When the Seiberg dual is taken for the second gauge group factor by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 3.3. The gauge group is given by

$$SU(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + N_{c,1} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 8. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + g_2 \tilde{X}_3 \tilde{q}_2 + \tilde{g}_2 q_2 X_3 + \Phi_3 g_2 \tilde{g}_2 \right) + m_2 M_2.$$

When the Seiberg dual is taken for the second gauge group factor with different brane motion by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 3.4. The gauge group is given by

$$SU(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + N_{c,1} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n - 3)$  NS-branes,  $(n - 3)$  sets of D6-branes and  $(n - 3)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 9. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + f_1 X_1 q_2 + \tilde{f}_1 \tilde{q}_2 \tilde{X}_1 + \Phi_1 f_1 \tilde{f}_1 \right) + m_2 M_2.$$

- When the dual magnetic theory contains  $SU(\tilde{N}_{c,i})$  where  $3 \leq i \leq n - 1$

When the Seiberg dual is taken for the middle gauge group factor by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i - 1, i + 1, \dots, n$ , one follows the procedure given in the subsection 2.3 of [18]. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i - 2)$  NS-branes,  $(i - 2)$  sets of D6-branes and  $(i - 2)$  sets of D4-branes are present between the  $NS5'_M$ -brane and the NS5'-brane and the extra  $(n - i - 1)$  NS-branes,  $(n - i - 1)$  sets of D6-branes and  $(n - i - 1)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 8. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + g_i \tilde{X}_{i+1} \tilde{q}_i + \tilde{g}_i q_i X_{i+1} + \Phi_{i+1} g_i \tilde{g}_i \right) + m_i M_i.$$

When the Seiberg dual is taken for the middle gauge group factor with different brane motion by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i - 1, i + 1, \dots, n$ , one follows the procedure given in the subsection 2.4 of [18]. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i - 2)$  NS-branes,  $(i - 2)$  sets of D6-branes and  $(i - 2)$  sets of D4-branes are present between the  $NS5'_M$ -brane and the NS5'-brane and the extra  $(n - i - 1)$  NS-branes,  $(n - i - 1)$  sets of D6-branes and  $(n - i - 1)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 9. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + f_{i-1} X_{i-1} q_i + \tilde{f}_{i-1} \tilde{q}_i \tilde{X}_{i-1} + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \right) + m_i M_i.$$



- When the dual magnetic theory contains  $SU(\tilde{N}_{c,n})$

When the Seiberg dual is taken for the last gauge group factor by assuming that  $\Lambda_n \gg \Lambda_i$  where  $i = 1, 2, \dots, (n-1)$ , one follows the procedure given in the subsection 3.5. The gauge group is given by

$$SU(N_{c,1}) \times \cdots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present between the  $NS5'_M$ -brane and the  $NS5_L$ -brane of Figure 10. The magnetic superpotential can be written as

$$W_{dual} = \left( M_n q_n \tilde{q}_n + g_{n-1} X_{n-1} q_n + \tilde{g}_{n-1} \tilde{q}_n \tilde{X}_{n-1} + \Phi_{n-1} g_{n-1} \tilde{g}_{n-1} \right) + m_n M_n.$$

## 4 Meta-stable brane configurations of other multiple product gauge theories

After we describe the electric brane configuration as we did previously, we present the three magnetic brane configurations, and then the nonsupersymmetric meta-stable brane configurations are found. The generalization to multiple product gauge groups is discussed.

### 4.1 Electric theory

Let us describe the gauge theory with triple product gauge groups  $SO(N_c) \times SU(N'_c) \times SU(N''_c)$  and the matter contents are

- $2N_f$ -chiral multiplets  $Q$  are in the representation  $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$
- $N'_f$ -chiral multiplets  $Q'$  are in the representation  $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ , and  $N'_f$ -chiral multiplets  $\tilde{Q}'$  are in the representation  $(\mathbf{1}, \overline{\mathbf{N}'_c}, \mathbf{1})$
- $N''_f$ -chiral multiplets  $Q''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c)$ , and  $N''_f$ -chiral multiplets  $\tilde{Q}''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}''_c})$
- The flavor-singlet field  $F$  is in the bifundamental representation  $(\mathbf{N}_c, \overline{\mathbf{N}'_c}, \mathbf{1})$ , and its conjugate field  $\tilde{F}$  is in the bifundamental representation  $(\mathbf{N}_c, \mathbf{N}'_c, \mathbf{1})$
- The flavor-singlet field  $G$  is in the bifundamental representation  $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}''_c})$ , and its conjugate field  $\tilde{G}$  is in the bifundamental representation  $(\mathbf{1}, \overline{\mathbf{N}'_c}, \mathbf{N}''_c)$

If we put to  $Q'', \tilde{Q}'', G$ , and  $\tilde{G}$  zero, then this becomes the product gauge group theory with fundamentals and bifundamentals [16, 17]. On the other hand, if we ignore  $Q, F$ , and  $\tilde{F}$ , then this theory is given by [13].

In the electric theory, the coefficient of the beta function of the first gauge group factor is  $b_{SO(N_c)} = 3(N_c - 2) - 2N_f - 2N'_c$  and similarly the coefficient of the beta function of the second gauge group factor is  $b_{SU(N'_c)} = 3N'_c - N'_f - N_c - N''_c$  and the coefficient of the beta function of the third gauge group factor is  $b_{SU(N''_c)} = 3N''_c - N''_f - N'_c$ . We'll see how these coefficients change in the magnetic theory. We denote the strong coupling scales for  $SO(N_c)$  as  $\Lambda_1$ , for  $SU(N'_c)$  as  $\Lambda_2$  and for  $SU(N''_c)$  as  $\Lambda_3$  respectively.

From the electric superpotential

$$W_{elec} = \left( \mu A^2 + QA\tilde{Q} + \tilde{F}AF + A_s^2 + QA_s\tilde{Q} + \tilde{F}A_sF + \mu' A'^2 + Q'A'\tilde{Q}' + \tilde{F}A'F \right. \\ \left. + \tilde{G}A'G + \mu'' A''^2 + Q''A''\tilde{Q}'' + \tilde{G}A''G \right) + mQQ + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''$$

one integrates out the adjoint fields  $A$  for  $SO(N_c)$ ,  $A'$  for  $SU(N'_c)$  and  $A''$  for  $SU(N''_c)$  and the symmetric field  $A_s$  for  $SO(N_c)$  and taking  $\mu, \mu'$  and  $\mu''$  to infinity limit which is equivalent to take any two NS-branes be perpendicular to each other, the mass-deformed electric superpotential becomes  $W_{elec} = mQQ + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''$ .

The type IIA brane configuration for this mass-deformed theory can be described by as follows. The  $N_c$ -color D4-branes (01236) are suspended between the  $NS5_L$ -brane (012345) and its mirror  $\overline{NS5_L}$ -brane together with  $N_f$  D6-branes (0123789) which have nonzero  $v$  direction. The  $NS5'$ -brane is located at the right hand side of the  $NS5_L$ -brane along the positive  $x^6$  direction and there exist  $N'_c$ -color D4-branes suspended between them, with  $N'_f$  D6-branes which have nonzero  $v$  direction. Moreover, the  $NS5_R$ -brane is located at the right hand side of the  $NS5'$ -brane along the positive  $x^6$  direction and there exist  $N''_c$ -color D4-branes suspended between them, with  $N''_f$  D6-branes which have nonzero  $v$  direction. There exists an orientifold 6-plane (0123789) at the origin  $x^6 = 0$  and it acts as  $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ . Then the mirrors of above branes appear in the negative  $x^6$  region and are denoted by bar on the corresponding branes. From the left to the right, there are  $\overline{NS5_R}$ -,  $\overline{NS5'}$ -,  $\overline{NS5_L}$ -,  $NS5_L$ -,  $NS5'$ -, and  $NS5_R$ -branes.

We summarize the  $\mathcal{N} = 1$  supersymmetric electric brane configuration in type IIA string theory as follows:

- Four NS5-branes in (012345) directions.
- Two NS5'-branes in (012389) directions.
- Two sets of  $N_c(N'_c)[N''_c]$ -color D4-branes in (01236) directions.
- Two sets of  $N_f(N'_f)[N''_f]$  D6-branes in (0123789) directions.
- $O6^+$ -plane in (0123789) directions with  $x^6 = 0$

Now we draw this electric brane configuration in Figure 11 and we put the coincident  $N_f(N'_f)[N''_f]$  D6-branes with positive  $x^6$  in the nonzero  $v$  direction in general. This brane

configuration can be obtained from the brane configuration of [16, 17] by adding the two outer NS5-branes(i.e.,  $\overline{NS5_R}$ -brane and  $NS5_R$ -brane), two sets of  $N_c''$  D4-branes and two sets of  $N_f''$  D6-branes or from the one of [13] with the gauge theory of triple product gauge groups by adding  $O6^+$ -plane and the extra NS-branes, D4-branes and D6-branes. The brane configuration for the single gauge group  $SO(N_c)$  was studied in [12]. Then the mirrors with negative  $x^6$  can be constructed by using the action of  $O6$ -plane and are located at the positions by changing (456) directions for the original branes with minus signs.

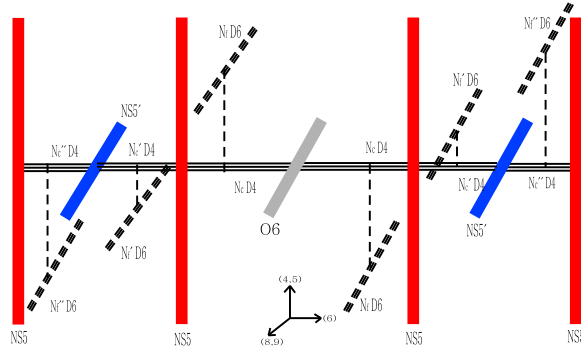


Figure 11: The  $\mathcal{N} = 1$  supersymmetric electric brane configuration with  $SO(N_c) \times SU(N_c') \times SU(N_c'')$  gauge group with flavors  $Q(Q')[Q'']$  and  $(\tilde{Q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(G), \tilde{F}(\tilde{G})$ . The  $O6^+$ -plane is located at the origin  $x^6 = 0$ . The two NS5-branes with positive  $x^6$  coordinates are denoted by  $NS5_{L,R}$ -branes.

## 4.2 Magnetic theory with dual for first gauge group

In this case, there is no extra NS-brane which should be present in order to construct the recombination of flavor D4-branes and splitting procedure for meta-stable brane configuration. Although the magnetic dual theory is present, there is no nonsupersymmetric meta-stable brane configuration.

## 4.3 Magnetic theory with dual for second gauge group

By moving the NS5'-brane in Figure 11 with massive  $N_f'$  flavors and massless  $N_f$  and  $N_f''$  flavors to the left all the way past the  $NS5_L$ -brane, one arrives at the Figure 12A. The linking number of NS5'-brane from Figure 12A is  $L_5 = -\frac{N_f'}{2} + \tilde{N}_c' - N_c$  while the linking number of NS5'-brane from Figure 11 is  $L_5 = \frac{N_f'}{2} + N_c'' - N_c'$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c' = N_f' + N_c'' + N_c - N_c'.$$

Let us draw this magnetic brane configuration in Figure 12A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore  $NS5_R$ -brane,  $N''_f$  D6-branes and  $N'_c$  D4-branes (detaching these branes from Figure 12A), then this brane configuration looks similar to the Figure 6 of [17] for the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SO(N_c) \times SU(\tilde{N}'_c = N'_f + N_c - N'_c)$  with fundamentals, bifundamentals, and singlets.

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes (connecting between D6-branes and  $NS5_L$ -brane) with the same number of color D4-branes (connecting between  $NS5'_L$ -brane and  $NS5_L$ -brane) and push them in  $+v$  direction from Figure 12A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane.

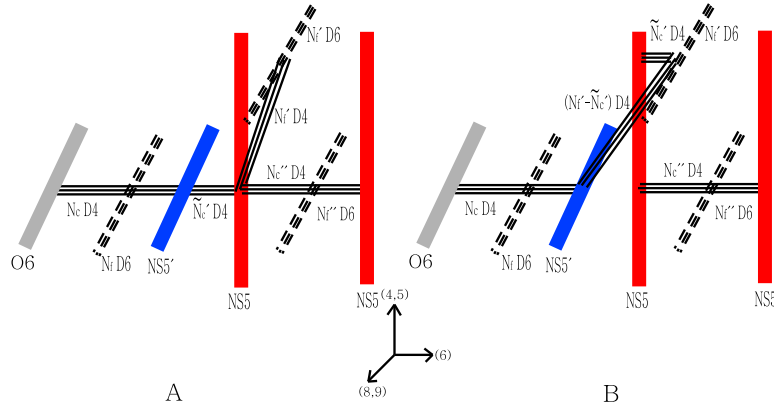


Figure 12: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SO(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q')[Q'']$  and  $(\tilde{q}')[\tilde{Q}']$  for each gauge group and bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ , and gauge singlets in Figure 12A. In Figure 12B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $(\tilde{Q}'')$  is given. Compared with the Figure 12A, there exists a misalignment of the flavor D4-branes.

In the dual theory, the coefficient of the beta function of the first gauge group factor is  $b_{SO(N_c)}^{mag} = 3(N_c - 2) - 2N_f - 2\tilde{N}'_c$  and the coefficient of the beta function of the second gauge group factor is  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$  and the coefficient of the beta function of the third gauge group factor is  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c - N'_f - N''_c$ .

The superpotential from [17] is

$$W_{dual} = (M' q' \tilde{q}' + m' M') + X'' \tilde{g} q' + \tilde{X}'' \tilde{q}' g + \Phi'' g \tilde{g}$$

and one sees that  $q' \tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term

equations. Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'} e^\phi \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'} e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X'' \rangle = \langle \tilde{X}'' \rangle = 0. \end{aligned}$$

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.

Then the gauge group and matter contents we consider are summarized as follows:

gauge group :	$SO(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$	
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 2N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X''_{n'} \equiv) \tilde{G} Q' \oplus G \tilde{Q}' (\equiv \tilde{X}''_{\tilde{n}'})$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q' \tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi'' \equiv) G \tilde{G}$	$(\mathbf{1}, \mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The nonsupersymmetric minimal energy brane configuration Figure 12B with a replacement  $N'_f$  D6-branes by the NS5'-brane (neglecting the  $NS5_R$ -brane,  $N''_f$  D6-branes,  $N_f$  D6-branes, and  $N''_c$  D4-branes) leads to the Figure 15B of [27].

In [5], the Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f + N'_f + N''_f} v^{2N_f + 2N''_f + 4} (v^2 - m'^2)^{N'_f}$  was found. Since we are dealing with six NS5-branes, the magnetic M5-brane configuration in Figure 12 with equal mass for  $q'$  and  $\tilde{q}'$  and massless for  $Q$  and  $Q''(\tilde{Q}'')$  can be characterized by the following sixth order equation for  $t$  as follows:

$$\begin{aligned} & t^6 + \left[ v^{N''_c} \right] t^5 + \left[ v^{\tilde{N}'_c + N''_f} (v + m')^{N'_f} \right] t^4 + \left[ v^{N_c + 2N''_f} (v + m')^{2N'_f} \right] t^3 \\ & + \left[ (-1)^{\tilde{N}'_c + N_f} v^{\tilde{N}'_c + 3N''_f + 2N_f + 4} (v + m')^{3N'_f} \right] t^2 + \left[ (-1)^{N''_c} v^{N''_c + 4N''_f + 4N_f + 8} (v + m')^{4N'_f} \right] t \\ & + \left[ (-1)^{N_f + N'_f + N''_f} v^{12 + 6N_f + 6N''_f} (v + m')^{5N'_f} (v - m')^{N'_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned}
w &\rightarrow 0, & y &\sim v^{N_c''} + \dots & \overline{NS5_R} \text{ asymptotic region,} \\
w &\rightarrow 0, & y &\sim v^{N_f'' + N_f' + \tilde{N}_c' - N_c''} + \dots & \overline{NS5_L} \text{ asymptotic region,} \\
w &\rightarrow 0, & y &\sim v^{-\tilde{N}_c' + N_f'' + 2N_f + N_f' + N_c'' + 4} + \dots & NS5_L \text{ asymptotic region,} \\
w &\rightarrow 0, & y &\sim v^{2N_f'' + 2N_f + 2N_f' - N_c'' + 4} + \dots & NS5_R \text{ asymptotic region.}
\end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned}
v &\rightarrow -m', & y &\sim w^{-\tilde{N}_c' + N_c + N_f'' + N_f'} + \dots & \overline{NS5'} \text{ asymptotic region,} \\
v &\rightarrow +m', & y &\sim w^{\tilde{N}_c' - N_c + N_f'' + 2N_f + N_f' + 4} + \dots & NS5' \text{ asymptotic region.}
\end{aligned}$$

#### 4.4 Magnetic theory with dual for second gauge group

By moving the  $NS5_L$ -brane in Figure 11 with massive  $N_f'$  D6-branes to the right all the way past the  $NS5'$ -brane, one arrives at the Figure 13A. The linking number of  $NS5_L$ -brane from Figure 13A is  $L_5 = \frac{N_f'}{2} - \tilde{N}_c' + N_c''$  and the linking number of  $NS5_L$ -brane from the Figure 11 is  $L_5 = -\frac{N_f'}{2} + N_c' - N_c$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c' = N_f' + N_c'' + N_c - N_c'.$$

Let us draw this magnetic brane configuration in Figure 13A and recall that we put the coincident  $N_f'$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N_f''$  D6-branes at  $v = 0$ . If we ignore  $NS5_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes(detaching these branes from Figure 13A), then this brane configuration leads to the Figure 6 of [17] for the standard  $\mathcal{N} = 1$  magnetic gauge theory  $SO(N_c) \times SU(\tilde{N}_c' = N_f' + N_c - N_c')$  with fundamentals, bifundamentals, and singlets.

Now let us recombine  $\tilde{N}_c'$  flavor D4-branes among  $N_f'$  flavor D4-branes(connecting between D6-branes and  $NS5'$ -brane) with the same number of color D4-branes(connecting between  $NS5'$ -brane and  $NS5_L$ -brane) and push them in  $+v$  direction from Figure 13A. For the flavor D4-branes, we are left with only  $(N_f' - \tilde{N}_c') = N_c' - N_c'' - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'$ -brane.

In the dual theory, the coefficient of the beta function of the first gauge group factor is  $b_{SO(N_c)}^{mag} = 3(N_c - 2) - 2N_f - 2\tilde{N}_c' - 2N_f' - 2N_c$  and the coefficient of the beta function of the second gauge group factor is  $b_{SU(\tilde{N}_c')}^{mag} = 3\tilde{N}_c' - N_f' - N_c - N_c''$  and the coefficient of the beta function of the third gauge group factor is  $b_{SU(N_c'')}^{mag} = 3N_c'' - N_f'' - \tilde{N}_c'$ .

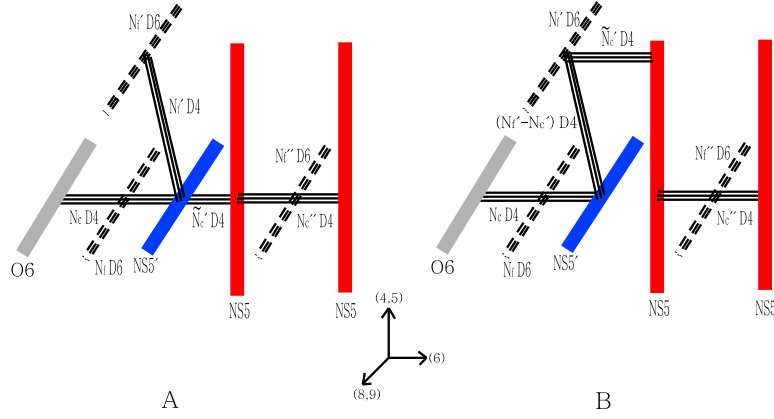


Figure 13: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SO(N_c) \times SU(\tilde{N}'_c = N'_f + N_c + N''_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q)[Q'']$  and  $(\tilde{q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $f(G)$  and  $\tilde{f}(\tilde{G})$ , and gauge singlets in Figure 13A. In Figure 13B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $(\tilde{Q}'')$  is given.

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SO(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 2N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X_{\tilde{n}'} \equiv) FQ' \oplus \tilde{F}\tilde{Q}' (\equiv \tilde{X}_{n'})$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\square, \mathbf{1}, \mathbf{1}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi \equiv) F\tilde{F}$	$(\mathbf{adj}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{symm}, \mathbf{1}, \mathbf{1})$

From the superpotential from [17]

$$W_{dual} = (M'q'\tilde{q}' + m'M') + Xfq' + \tilde{X}\tilde{q}'\tilde{f} + \Phi f\tilde{f} + Q\Phi Q$$

one sees that  $q'\tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term

equations. Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'} e^{\phi} \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'} e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X \rangle = \langle \tilde{X} \rangle = \langle Q \rangle = 0. \end{aligned}$$

Let us expand around a point on the vacua, as done in [1]. Then the remaining relevant terms of superpotential are given by  $W_{dual}^{rel} = M'_0 (\delta\varphi \delta\tilde{\varphi} + m') + \delta Z \delta\varphi \tilde{q}'_0 + \delta\tilde{Z} q'_0 \delta\tilde{\varphi}$  by following the similar fluctuations for the various fields as in [7]. Note that there exist also four kinds of terms, the vacuum  $\langle q' \rangle$  multiplied by  $\delta f \delta X$ , the vacuum  $\langle \tilde{q}' \rangle$  multiplied by  $\delta\tilde{X} \delta\tilde{f}$ , the vacuum  $\langle \Phi \rangle$  multiplied by  $\delta f \delta\tilde{f}$ , and the vacuum  $\langle \Phi \rangle$  multiplied by  $\delta Q \delta\tilde{Q}$ . However, by redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [22], one gets that  $m_{M'_0}^2$  will contain  $(\log 4 - 1) > 0$  implying that these are stable.

The nonsupersymmetric minimal energy brane configuration Figure 13B (neglecting the  $NS5_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes) looks similar to the Figure 6 of [17].

The Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f + N'_f + N''_f} v^{2N_f + 2N''_f + 4} (v^2 - m'^2)^{N'_f}$  was found. Since we are dealing with six NS-branes, the magnetic M5-brane configuration in Figure 13 with equal mass for  $q$  and  $\tilde{q}'$  and massless for  $Q(Q'')$  and  $\tilde{Q}''$  can be characterized by the following sixth order equation for  $t$  as follows:

$$\begin{aligned} &t^6 + \left[ v^{N''_c} \right] t^5 + \left[ v^{\tilde{N}'_c + N''_f} \right] t^4 + \left[ v^{N_c + 2N''_f} \right] t^3 \\ &+ \left[ (-1)^{\tilde{N}'_c + N_f + N'_f} v^{\tilde{N}'_c + 3N''_f + 2N_f + 4} (v^2 - m'^2)^{N'_f} \right] t^2 + \left[ (-1)^{N''_c} v^{N''_c + 4N''_f + 4N_f + 8} (v^2 - m'^2)^{2N'_f} \right] t \\ &+ \left[ (-1)^{N_f + N'_f + N''_f} v^{12 + 6N_f + 6N''_f} (v^2 - m'^2)^{3N'_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, \quad y \sim v^{N''_c} + \dots \quad \overline{NS5_R} \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{N''_f + \tilde{N}'_c - N''_c} + \dots \quad \overline{NS5_L} \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{-\tilde{N}'_c + N''_f + N''_c + 2N'_f + 2N_f + 4} + \dots \quad NS5_L \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{2N''_f + 2N_f + 2N'_f - N''_c + 4} + \dots \quad NS5_R \text{ asymptotic region.} \end{aligned}$$



2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v \rightarrow -m', \quad y &\sim w^{-\tilde{N}_c' + N_c + N_f''} + \dots \quad \overline{NS5'} \text{ asymptotic region,} \\ v \rightarrow +m', \quad y &\sim w^{\tilde{N}_c' - N_c + N_f'' + 2N_f + 2N_f' + 4} + \dots \quad NS5' \text{ asymptotic region.} \end{aligned}$$

## 4.5 Magnetic theory with dual for third gauge group

By moving the  $NS5_R$ -brane with massive  $N_f''$  D6-branes to the left all the way past the  $NS5'$ -brane, one arrives at the Figure 14A. The linking number of  $NS5_R$ -brane from Figure 14A is given by  $L_5 = \frac{N_f''}{2} - \tilde{N}_c''$  and the linking number of  $NS5_R$ -brane from Figure 11 is  $L_5 = -\frac{N_f''}{2} + N_c'' - N_c'$ . From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c'' = N_f'' + N_c' - N_c''.$$

Let us draw this magnetic brane configuration in Figure 14A and recall that we put the coincident  $N_f''$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q'$  by putting  $N_f$  and  $N_f'$  D6-branes at  $v = 0$ .

Now let us recombine  $\tilde{N}_c''$  flavor D4-branes among  $N_f''$  flavor D4-branes (connecting between D6-branes and  $NS5'$ -brane) with the same number of color D4-branes (connecting between  $NS5_R$ -brane and  $NS5'$ -brane) and push them in  $+v$  direction from Figure 14A. For the flavor D4-branes, we are left with only  $(N_f'' - \tilde{N}_c'') = N_c'' - N_c'$  flavor D4-branes connecting between D6-branes and  $NS5'$ -brane.

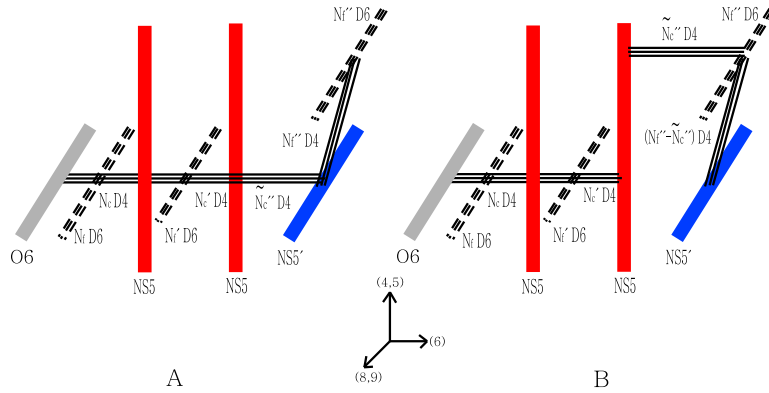


Figure 14: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $SO(N_c) \times SU(N_c') \times SU(\tilde{N}_c'' = N_f'' + N_c' - N_c'')$  gauge group with fundamentals  $Q(Q')[q'']$  and  $(\tilde{Q}')[\tilde{q}'']$  for each gauge group, bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ , and gauge singlets in Figure 14A. In Figure 14B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q')$  and  $(\tilde{Q}')$  is given.

The coefficient of the beta function of first gauge group factor is  $b_{SO(N_c)}^{mag} = 3(N_c - 2) - 2N_f - 2N'_c = b_{SO(N_c)}$  and the coefficient of the beta function of second gauge group factor is  $b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - N_c - \tilde{N}''_c - N''_f - N'_c$  and the coefficient of the beta function of third gauge group factor is  $b_{SU(\tilde{N}''_c)}^{mag} = 3\tilde{N}''_c - N''_f - N'_c$ . Since  $b_{SU(N'_c)} - b_{SU(\tilde{N}''_c)}^{mag} > 0$ ,  $SU(N'_c)$  is more asymptotically free than  $SU(\tilde{N}''_c)^{mag}$ .

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$SO(N_c) \times SU(N'_c) \times SU(\tilde{N}''_c)$
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 2N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \bar{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$q''_{f''} \oplus \tilde{q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \bar{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \bar{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \bar{\square}) \oplus (\mathbf{1}, \bar{\square}, \square)$
	$(X'_{n''} \equiv) GQ'' \oplus \tilde{G}\tilde{Q}'' (\equiv \tilde{X}'_{\tilde{n}''})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \bar{\square}, \mathbf{1}) \quad (n'', \tilde{n}'' = 1, \dots, N''_f)$
	$(M''_{f'', \tilde{g}''} \equiv) Q''\tilde{Q}''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f'', \tilde{g}'' = 1, \dots, N''_f)$
	$(\Phi' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

From the superpotential

$$W_{dual} = (M''q''\tilde{q}'' + m''M'') + X'gq'' + \tilde{X}'\tilde{q}''\tilde{g} + \Phi'g\tilde{g}$$

$q''\tilde{q}''$  has rank  $\tilde{N}''_c$  while  $m''$  has a rank  $N''_f$ . The derivative of the superpotential  $W_{dual}$  with respect to  $M''$  cannot be satisfied if the rank  $N''_f$  exceeds  $\tilde{N}''_c$  and the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then the solutions can be written as

$$\begin{aligned} \langle q'' \rangle &= \begin{pmatrix} \sqrt{m''}e^{\phi}\mathbf{1}_{\tilde{N}''_c} \\ 0 \end{pmatrix}, \langle \tilde{q}'' \rangle = \begin{pmatrix} \sqrt{m''}e^{-\phi}\mathbf{1}_{\tilde{N}''_c} & 0 \end{pmatrix}, \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M''_0\mathbf{1}_{N''_f - \tilde{N}''_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

It turns out that states are stable by realizing the mass of  $m_{M''_0}^2$  positive, by expanding the fields around the vacua.

The nonsupersymmetric minimal energy brane configuration Figure 14B with a replacement  $N''_f$  D6-branes by the NS5'-brane (neglecting the  $NS5_L$ -brane,  $N_f$  D6-branes,  $N'_f$  D6-branes, and  $N_c$  D4-branes) looks similar to the Figure 16B of [27]. The position of NS5'-brane is different from each other.

The Riemann surface describing a set of NS5-branes with D4-branes suspended between them and in a background space of  $xt = (-1)^{N_f+N'_f+N''_f} v^{2N_f+2N'_f+4} (v^2 - m''^2)^{N''_f}$  was found. Since we are dealing with six NS-branes, the magnetic M5-brane configuration in Figure 14 with equal mass for  $q''$  and  $\tilde{q}''$  and massless for  $Q$  and  $Q'(\tilde{Q}')$  can be characterized by the following sixth order equation for  $t$  as follows:

$$\begin{aligned} & t^6 + \left[ v^{\tilde{N}''_c} \right] t^5 + \left[ v^{N'_c} (v + m'')^{N''_f} \right] t^4 + \left[ v^{N_c+N'_f} (v + m'')^{2N''_f} \right] t^3 \\ & + \left[ (-1)^{N'_c+N_f} v^{N'_c+2N'_f+2N_f+4} (v + m'')^{3N''_f} \right] t^2 + \left[ (-1)^{\tilde{N}''_c+N'_f} v^{\tilde{N}''_c+4N'_f+4N_f+8} (v + m'')^{4N''_f} \right] t \\ & + \left[ (-1)^{N_f+N''_f} v^{12+6N'_f+6N_f} (v - m'')^{N''_f} (v + m'')^{5N''_f} \right] = 0. \end{aligned}$$

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w & \rightarrow 0, \quad y \sim v^{N'_c+N''_f-\tilde{N}''_c} + \dots \quad \overline{NS5_R} \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N''_f+N'_f+N_c-N'_c} + \dots \quad \overline{NS5_L} \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N'_c+N''_f+2N_f+N'_f-N_c+4} + \dots \quad NS5_L \text{ asymptotic region,} \\ w & \rightarrow 0, \quad y \sim v^{N''_f+2N_f+2N'_f-N'_c+\tilde{N}''_c+4} + \dots \quad NS5_R \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v & \rightarrow -m'', \quad y \sim w^{\tilde{N}''_c} + \dots \quad \overline{NS5'} \text{ asymptotic region,} \\ v & \rightarrow +m'', \quad y \sim w^{-\tilde{N}''_c+2N''_f+2N'_f+2N_f+4} + \dots \quad NS5' \text{ asymptotic region.} \end{aligned}$$

## 4.6 Magnetic theories for the multiple product gauge groups

Now one can generalize the method for the triple product gauge groups to the finite  $n$ -multiple product gauge groups characterized by

$$SO(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})$$

with the matter, the  $(n-1)$  bifundamentals  $(\square_1, \overline{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_{n-1}, \overline{\square}_n)$ , their complex conjugate  $(n-1)$  fields  $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \overline{\square}_{n-1}, \square_n)$ , linking the gauge groups together,  $(n-1)$ -fundamentals  $(\mathbf{1}_1, \square_2, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_n)$ , and  $(n-1)$ -antifundamentals  $(\mathbf{1}_1, \overline{\square}_2, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \overline{\square}_n)$  and one-vector in the representation  $(\square_1, \dots, \mathbf{1}_n)$ . Then the mass-deformed superpotential can be written as

$W_{elec} = m_1 Q_1 Q_1 + \sum_{i=2}^n m_i Q_i \tilde{Q}_i$ . The brane configuration can be constructed from Figure 11 by adding  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes to the right of  $NS5_R$ -brane (and its mirrors) leading to the fact that any two neighboring NS-branes should be perpendicular to each other.

There exist  $(2n-3)$  magnetic theories and they can be classified as follows.

- When the dual magnetic gauge group is  $SO(\tilde{N}_{c,1})$

There is no nonsupersymmetric meta-stable brane configuration.

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,2})$

When the Seiberg dual is taken for the second gauge group factor by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 4.3. The gauge group is given by

$$SO(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,1} + N_{c,3} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 12. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + g_2 \tilde{X}_3 \tilde{q}_2 + \tilde{g}_2 q_2 X_3 + \Phi_3 g_2 \tilde{g}_2 \right) + m_2 M_2.$$

By computing the contribution for the one loop as in the subsection 4.3, the vacua are stable and the asymptotic behavior of  $2n$  NS-branes can be obtained.

When the Seiberg dual is taken for the second gauge group factor with different brane motion by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 4.4. The gauge group is given by

$$SO(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + N_{c,1} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 13. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + f_1 X_1 q_2 + \tilde{f}_1 \tilde{q}_2 \tilde{X}_1 + \Phi_1 f_1 \tilde{f}_1 + Q \Phi_1 Q \right) + m_2 M_2.$$

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,i})$  where  $3 \leq i \leq n-1$

When the Seiberg dual is taken for the middle gauge group factor by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i-1, i+1, \dots, n$ , one follows the procedure given in the subsection 2.3 of [18]. The gauge group is given by

$$SO(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i - 2)$  NS-branes,  $(i - 2)$  sets of D6-branes and  $(i - 2)$  sets of D4-branes are present between O6-plane and the NS5'-brane in Figure 12 and the extra  $(n - i - 1)$  NS-branes,  $(n - i - 1)$  sets of D6-branes and  $(n - i - 1)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 12. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + g_i \tilde{X}_{i+1} \tilde{q}_i + \tilde{g}_i q_i X_{i+1} + \Phi_{i+1} g_i \tilde{g}_i \right) + m_i M_i.$$

When the Seiberg dual is taken for the middle gauge group factor with different brane motion by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i - 1, i + 1, \dots, n$ , one follows the procedure given in the subsection 2.4 of [18]. The gauge group is given by

$$SO(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i - 2)$  NS-branes,  $(i - 2)$  sets of D6-branes and  $(i - 2)$  sets of D4-branes are present between O6-plane and the NS5'-brane in Figure 13 and the extra  $(n - i - 1)$  NS-branes,  $(n - i - 1)$  sets of D6-branes and  $(n - i - 1)$  sets of D4-branes are present at the right hand side of the  $NS5_R$ -brane of Figure 13. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + f_{i-1} X_{i-1} q_i + \tilde{f}_{i-1} \tilde{q}_i \tilde{X}_{i-1} + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \right) + m_i M_i.$$

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,n})$

When the Seiberg dual is taken for the last gauge group factor by assuming that  $\Lambda_n \gg \Lambda_i$  where  $i = 1, 2, \dots, (n - 1)$ , one follows the procedure given in the subsection 4.5. The gauge group is given by

$$SO(N_{c,1}) \times \dots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n - 3)$  NS-branes,  $(n - 3)$  sets of D6-branes and  $(n - 3)$  sets of D4-branes are present between O6-plane and the  $NS5_L$ -brane of Figure 14. The magnetic superpotential can be written as

$$W_{dual} = \left( M_n q_n \tilde{q}_n + g_{n-1} X_{n-1} q_n + \tilde{g}_{n-1} \tilde{q}_n \tilde{X}_{n-1} + \Phi_{n-1} g_{n-1} \tilde{g}_{n-1} \right) + m_n M_n.$$

## 5 More meta-stable brane configurations of other multiple product gauge theories

After we describe the electric brane configuration, we present the three magnetic brane configurations, and then the nonsupersymmetric meta-stable brane configurations are found. The case of multiple product gauge groups is also discussed.

## 5.1 Electric theory

Let us describe the gauge theory with triple product gauge groups  $Sp(N_c) \times SU(N'_c) \times SU(N''_c)$ . The matter contents are

- $2N_f$ -chiral multiplets  $Q$  are in the representation  $(2\mathbf{N}_c, \mathbf{1}, \mathbf{1})$
- $N'_f$ -chiral multiplets  $Q'$  are in the representation  $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ , and  $N'_f$ -chiral multiplets  $\tilde{Q}'$  are in the representation  $(\mathbf{1}, \overline{\mathbf{N}'_c}, \mathbf{1})$
- $N''_f$ -chiral multiplets  $Q''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c)$ , and  $N''_f$ -chiral multiplets  $\tilde{Q}''$  are in the representation  $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}''_c})$
- The flavor-singlet field  $F$  is in the bifundamental representation  $(2\mathbf{N}_c, \overline{\mathbf{N}'_c}, \mathbf{1})$ , and its conjugate field  $\tilde{F}$  is in the bifundamental representation  $(2\mathbf{N}_c, \mathbf{N}'_c, \mathbf{1})$
- The flavor-singlet field  $G$  is in the bifundamental representation  $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}''_c})$ , and its conjugate field  $\tilde{G}$  is in the bifundamental representation  $(\mathbf{1}, \overline{\mathbf{N}'_c}, \mathbf{N}''_c)$

If we put to  $Q'', \tilde{Q}'', G$ , and  $\tilde{G}$  zero, then this becomes the product gauge group theory with fundamentals and bifundamentals [16, 17]. On the other hand, if we ignore  $Q, F$ , and  $\tilde{F}$ , then this theory is given by [13].

The coefficient of the beta function of the first gauge group factor is  $b_{Sp(N_c)} = 3(N_c + 2) - 2N_f - 2N'_c$  and similarly the coefficient of the beta function of the second gauge group factor is  $b_{SU(N'_c)} = 3N'_c - N'_f - N_c - N''_c$  and finally the coefficient of the beta function of the third gauge group factor is  $b_{SU(N''_c)} = 3N''_c - N''_f - N'_c$ .

From the electric superpotential

$$\begin{aligned}
W_{elec} = & \left( \mu A^2 + QA\tilde{Q} + \tilde{F}AF + A_a^2 + QA_a\tilde{Q} + \tilde{F}A_aF + \mu' A'^2 + Q'A'\tilde{Q}' + \tilde{F}A'F \right. \\
& \left. + \tilde{G}A'G + \mu'' A''^2 + Q''A''\tilde{Q}'' + \tilde{G}A''G \right) + mQQ + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''
\end{aligned}$$

one integrates out the adjoint fields  $A$  for  $Sp(N_c)$ ,  $A'$  for  $SU(N'_c)$  and  $A''$  for  $SU(N''_c)$  and the antisymmetric field  $A_a$  for  $Sp(N_c)$  and taking  $\mu, \mu'$  and  $\mu''$  to infinity limit which is equivalent to take any two NS-branes be perpendicular to each other, the mass-deformed electric superpotential becomes  $W_{elec} = mQQ + m'Q'\tilde{Q}' + m''Q''\tilde{Q}''$ .

The type IIA brane configuration for this mass-deformed theory can be described by as follows. The  $2N_c$ -color D4-branes (01236) are suspended between the  $NS5'_L$ -brane (012389) and its mirror  $\overline{NS5'_L}$ -brane together with  $N_f$  D6-branes (0123789) which have nonzero  $v$  direction. The NS5-brane is located at the right hand side of the  $NS5'_L$ -brane along the positive  $x^6$  direction and there exist  $N'_c$ -color D4-branes suspended between them, with  $N'_f$  D6-branes which have nonzero  $v$  direction. Moreover, the  $NS5'_R$ -brane is located at the right hand side of the NS5-brane along the positive  $x^6$  direction and there exist  $N''_c$ -color D4-branes

suspended between them, with  $N_f''$  D6-branes which have nonzero  $v$  direction. There exists an orientifold 6-plane (0123789) at the origin  $x^6 = 0$  and it acts as  $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ . Then the mirrors of above branes appear in the negative  $x^6$  region and are denoted by bar on the corresponding branes. From the left to the right, there are  $\overline{NS5'_R}$ -,  $\overline{NS5}$ -,  $\overline{NS5'_L}$ -,  $NS5'_L$ -,  $NS5$ -, and  $NS5'_R$ -branes.

We summarize the  $\mathcal{N} = 1$  supersymmetric electric brane configuration in type IIA string theory as follows:

- Two NS5-branes in (012345) directions.
- Four NS5'-branes in (012389) directions.
- Two sets of  $N_c(N'_c)[N''_c]$ -color D4-branes in (01236) directions.
- Two sets of  $N_f(N'_f)[N''_f]$  D6-branes in (0123789) directions.
- $O6^-$ -plane in (0123789) directions with  $x^6 = 0$

Now we draw this electric brane configuration in Figure 15 and we put the coincident  $N_f(N'_f)[N''_f]$  D6-branes with positive  $x^6$  in the nonzero  $v$  direction in general. This brane configuration can be obtained from the brane configuration of [16, 17] by adding the two outer NS5'-branes(i.e.,  $\overline{NS5'_R}$ -brane and  $NS5'_R$ -brane), two sets of  $N''_c$  D4-branes and two sets of  $N''_f$  D6-branes or from the one of [13] with the gauge theory of triple product gauge groups by adding  $O6$ -plane and the extra NS-branes, D4-branes and D6-branes. The brane configuration for single gauge  $Sp(N_c)$  theory was presented in [12]. Then the mirrors with negative  $x^6$  can be constructed by using the action of  $O6$ -plane and are located at the positions by changing (456) directions of original branes with minus signs.

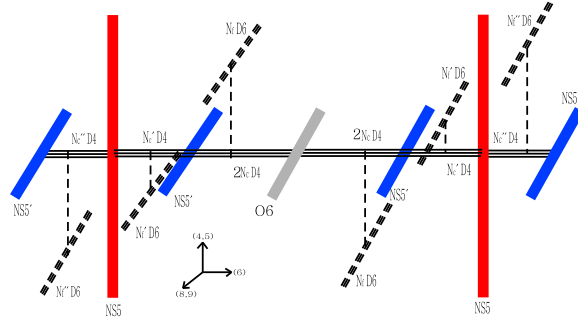


Figure 15: The  $\mathcal{N} = 1$  supersymmetric electric brane configuration with  $Sp(N_c) \times SU(N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(Q')[Q'']$  and  $(\tilde{Q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(G), \tilde{F}(\tilde{G})$ . The  $O6^-$ -plane is located at the origin  $x^6 = 0$ . The two NS5'-branes with positive  $x^6$  coordinates are denoted by  $NS5'_{L,R}$ -branes.

## 5.2 Magnetic theory with dual for first gauge group

In this case, there is no extra NS-brane which should be present in order to construct the recombination of flavor D4-branes and splitting procedure for meta-stable brane configuration. Although the magnetic dual theory is present, there is no nonsupersymmetric meta-stable brane configuration.

## 5.3 Magnetic theory with dual for second gauge group

By moving the NS5-brane in Figure 15 with massive  $N'_f$  D6-branes to the left all the way past the  $NS5'_L$ -brane, one arrives at the Figure 16A. The linking number of NS5-brane from Figure 16A is  $L_5 = -\frac{N'_f}{2} + \tilde{N}'_c - 2N_c$  while the linking number of NS5-brane from Figure 15 is  $L_5 = \frac{N'_f}{2} + N''_c - N'_c$ . From these two relations, one obtains the number of colors of dual magnetic theory  $\tilde{N}'_c = N'_f + N''_c + 2N_c - N'_c$ .

Let us draw this magnetic brane configuration in Figure 16A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore  $NS5'_R$ -brane,  $N''_f$  D6-branes and  $N'_c$  D4-branes(detaching these branes from Figure 16A), then this brane configuration looks similar to the Figure 7 of [17] for the standard  $\mathcal{N} = 1$  magnetic gauge theory  $Sp(N_c) \times SU(\tilde{N}'_c = N'_f + 2N_c - N'_c)$  with fundamentals, bifundamentals, and singlets.

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and  $NS5'_L$ -brane) with the same number of color D4-branes(connecting between NS5-brane and  $NS5'_L$ -brane) and push them in  $+v$  direction from Figure 16A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane.

The coefficient of the beta function of first gauge group factor is  $b_{Sp(N_c)}^{mag} = 3(N_c + 2) - 2N_f - 2\tilde{N}'_c$  and the coefficient of the beta function of second gauge group factor is  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$  and the coefficient of the beta function of third gauge group factor is  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c - N'_f - N''_c$ .

From the superpotential [17]  $W_{dual} = (M'q'\tilde{q}' + m'M') + X''\tilde{g}q' + \tilde{X}''\tilde{q}'g + \Phi''g\tilde{g}$  one sees that  $q'\tilde{q}'$  has rank  $\tilde{N}'_c$  while  $m'$  has a rank  $N'_f$ . If the rank  $N'_f$  exceeds  $\tilde{N}'_c$ , then the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then the solutions can be written as

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'}e^{\phi}\mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'}e^{-\phi}\mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X'' \rangle = \langle \tilde{X}'' \rangle = 0. \end{aligned}$$



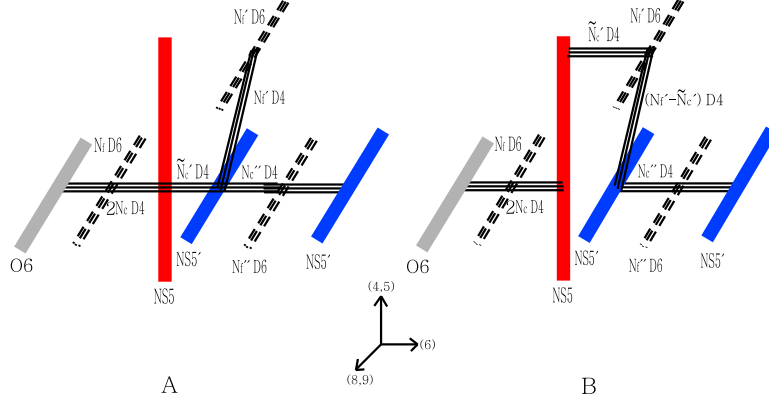


Figure 16: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $Sp(N_c) \times SU(\tilde{N}'_c = N'_f + 2N_c + N''_c - N'_c) \times SU(N''_c)$  gauge group with fundamentals  $Q(q')[Q'']$  and  $(\tilde{q}')[\tilde{Q}'']$  for each gauge group and bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ , and gauge singlets in Figure 16A. In Figure 16B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q'')$  and  $(\tilde{Q}'')$  is given.

By expanding the fields around the vacua and it turns out that states are stable by realizing the mass of  $m_{M'_0}^2$  positive.

Then the gauge group and matter contents we consider are summarized as follows:

gauge group :	$Sp(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$		
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1})$	$(f = 1, \dots, 2N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1})$	$(f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square})$	$(f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$	
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$	
	$(X''_{n'} \equiv) \tilde{G}Q' \oplus G\tilde{Q}' (\equiv \tilde{X}''_{\tilde{n}'})$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square})$	$(n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q' \tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi'' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$	

The nonsupersymmetric minimal energy brane configuration Figure 16B with a replacement  $N'_f$  D6-branes by the NS5'-brane (neglecting the  $NS5'_R$ -brane,  $N'_f$  D6-branes,  $N'_c$  D4-branes and  $N_f$  D6-branes) leads to the Figure 12B of [27].

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{-\tilde{N}'_c + 2N_c + N''_f + N'_f} + \dots & \overline{NS5} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{\tilde{N}'_c - 2N_c + N''_f + 2N_f + N'_f - 4} + \dots & NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow +m', & y &\sim w^{-\tilde{N}'_c + N''_f + 2N_f + N'_f + N''_c - 4} + \dots & NS5'_L \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{2N''_f + 2N_f + 2N'_f - N''_c - 4} + \dots & NS5'_R \text{ asymptotic region,} \\ v &\rightarrow -m', & y &\sim w^{N''_f + N'_f + \tilde{N}'_c - N''_c} + \dots & \overline{NS5}_L \text{ asymptotic region.} \\ v &\rightarrow -m', & y &\sim w^{N''_c} + \dots & \overline{NS5}_R \text{ asymptotic region.} \end{aligned}$$

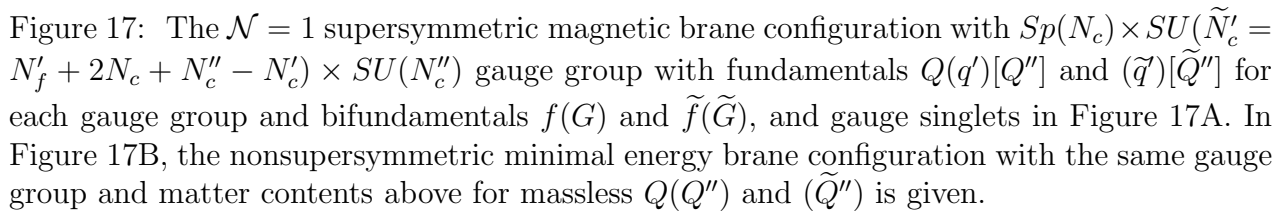
## 5.4 Magnetic theory with dual for second gauge group

By moving the  $NS5'_L$ -brane in Figure 15 with massive  $N'_f$  D6-branes to the right all the way past the NS5-brane, one arrives at the Figure 17A. The linking number of  $NS5'_L$ -brane from Figure 17A is  $L_5 = \frac{N'_f}{2} - \tilde{N}'_c + N''_c$  and the linking number of  $NS5'_L$ -brane from the Figure 15 is  $L_5 = -\frac{N'_f}{2} + N'_c - 2N_c$ . From these two relations, one obtains the number of colors of dual magnetic theory  $\tilde{N}'_c = N'_f + N''_c + 2N_c - N'_c$ .

Let us draw this magnetic brane configuration in Figure 17A and recall that we put the coincident  $N'_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q''(\tilde{Q}'')$  by putting  $N_f$  and  $N''_f$  D6-branes at  $v = 0$ . If we ignore  $NS5'_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes(detaching these branes from Figure 17A), then this brane configuration leads to the Figure 7 of [17] for the standard  $\mathcal{N} = 1$  magnetic gauge theory  $Sp(N_c) \times SU(\tilde{N}'_c = N'_f + 2N_c - N'_c)$  with fundamentals, bifundamentals, and singlets.

Now let us recombine  $\tilde{N}'_c$  flavor D4-branes among  $N'_f$  flavor D4-branes(connecting between D6-branes and NS5-brane) with the same number of color D4-branes(connecting between NS5-brane and  $NS5'_L$ -brane) and push them in  $+v$  direction from Figure 17A. For the flavor D4-branes, we are left with only  $(N'_f - \tilde{N}'_c) = N'_c - N''_c - N_c$  flavor D4-branes connecting between D6-branes and  $NS5'_L$ -brane.

The coefficient of the beta function of first gauge group factor is  $b_{Sp(N_c)}^{mag} = 3(N_c + 2) - 2N_f - 2\tilde{N}'_c - 2N'_f - 2N_c$  and the coefficient of the beta function of second gauge group factor is  $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c - N''_c$  and the coefficient of the beta function of third gauge group factor is  $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c$ .



gauge group :	$Sp(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$	
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 2N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X_{\tilde{n}'} \equiv) FQ' \oplus \tilde{F}\tilde{Q}' (\equiv \tilde{X}_{n'})$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\square, \mathbf{1}, \mathbf{1}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi \equiv) F\tilde{F}$	$(\mathbf{adj}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{asymm}, \mathbf{1}, \mathbf{1})$

$$W_{dual} = (M' q' \tilde{q}' + m' M') + X f q' + \tilde{X} \tilde{q}' \tilde{f} + \Phi f \tilde{f} + Q \Phi Q$$
$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m'} e^{\phi} \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m'} e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X \rangle = \langle \tilde{X} \rangle = \langle Q \rangle = 0. \end{aligned}$$

Let us expand around a point on the vacua, as done in [1]. Then the remaining relevant terms of superpotential are given by  $W_{dual}^{rel} = M'_0 (\delta\varphi \delta\tilde{\varphi} + m') + \delta Z \delta\varphi \tilde{q}'_0 + \delta\tilde{Z} q'_0 \delta\tilde{\varphi}$  by following the similar fluctuations for the various fields as in [7]. Note that there exist also four kinds of terms, the vacuum  $\langle q' \rangle$  multiplied by  $\delta f \delta X$ , the vacuum  $\langle \tilde{q}' \rangle$  multiplied by  $\delta\tilde{X} \delta\tilde{f}$ , the vacuum  $\langle \Phi \rangle$  multiplied by  $\delta f \delta\tilde{f}$ , and the vacuum  $\langle \Phi \rangle$  multiplied by  $\delta Q \delta\tilde{Q}$ . However, by redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [22], one gets that  $m_{M'_0}^2$  will contain  $(\log 4 - 1) > 0$  implying that these are stable.

The nonsupersymmetric minimal energy brane configuration Figure 17B (neglecting the  $NS5'_R$ -brane,  $N''_f$  D6-branes and  $N''_c$  D4-branes) leads to the Figure 7 of [17].

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, & y &\sim v^{-\tilde{N}'_c + 2N_c + N''_f} + \dots & \overline{NS5} \text{ asymptotic region,} \\ w &\rightarrow 0, & y &\sim v^{\tilde{N}'_c - 2N_c + N''_f + 2N_f + 2N'_f - 4} + \dots & NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow +m', & y &\sim w^{-\tilde{N}'_c + N''_f + N''_c + 2N'_f + 2N_f - 4} + \dots & NS5'_L \text{ asymptotic region,} \\ v &\rightarrow +m', & y &\sim w^{2N''_f + 2N_f + 2N'_f - N''_c - 4} + \dots & NS5'_R \text{ asymptotic region,} \\ v &\rightarrow -m', & y &\sim w^{N''_f + \tilde{N}'_c - N''_c} + \dots & \overline{NS5}'_L \text{ asymptotic region,} \\ v &\rightarrow -m', & y &\sim w^{N''_c} + \dots & \overline{NS5}'_R \text{ asymptotic region.} \end{aligned}$$

## 5.5 Magnetic theory with dual for third gauge group

By moving the NS5-brane with massive  $N''_f$  D6-branes to the right all the way past the  $NS5'_R$ -brane, one arrives at the Figure 18A. The linking number of NS5-brane from Figure 18A is given by  $L_5 = \frac{N''_f}{2} - \tilde{N}''_c$  and the linking number of NS5-brane from Figure 15 is  $L_5 = -\frac{N''_f}{2} + N''_c - N'_c$ . From these two relations, one obtains the number of colors of dual magnetic theory  $\tilde{N}''_c = N''_f + N'_c - N''_c$ .

Let us draw this magnetic brane configuration in Figure 18A and recall that we put the coincident  $N''_f$  D6-branes in the nonzero  $v$ -direction in the electric theory and consider massless flavors for  $Q$  and  $Q'$  by putting  $N_f$  and  $N'_f$  D6-branes at  $v = 0$ .

Now let us recombine  $\tilde{N}''_c$  flavor D4-branes among  $N''_f$  flavor D4-branes (connecting between D6-branes and  $NS5'_R$ -brane) with the same number of color D4-branes (connecting between

$NS5'_R$ -brane and NS5-brane) and push them in  $+v$  direction from Figure 18A. For the flavor D4-branes, we are left with only  $(N''_f - \tilde{N}''_c) = N''_c - N'_c$  flavor D4-branes connecting between D6-branes and  $NS5'_R$ -brane.

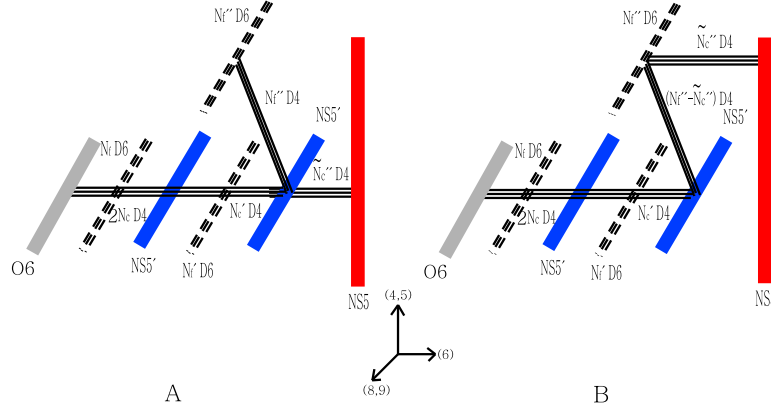


Figure 18: The  $\mathcal{N} = 1$  supersymmetric magnetic brane configuration with  $Sp(N_c) \times SU(N'_c) \times SU(\tilde{N}''_c = N''_f + N'_c - N''_c)$  gauge group with fundamentals  $Q(Q')[q'']$  and  $(\tilde{Q}')[\tilde{q}']$  for each gauge group, bifundamentals  $F(g)$  and  $\tilde{F}(\tilde{g})$ , and gauge singlets in Figure 18A. In Figure 18B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless  $Q(Q')$  and  $(\tilde{Q}')$  is given.

The coefficient of the beta function of first gauge group factor is  $b_{Sp(N_c)}^{mag} = 3(N_c + 2) - 2N_f - 2N'_c = b_{Sp(N_c)}$  and the coefficient of the beta function of second gauge group factor is  $b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - N_c - \tilde{N}''_c - N''_f - N'_c$  and the coefficient of the beta function of third gauge group factor is  $b_{SU(\tilde{N}''_c)}^{mag} = 3\tilde{N}''_c - N''_f - N'_c$ . Since  $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} > 0$ ,  $SU(N'_c)$  is more asymptotically free than  $SU(N'_c)^{mag}$ .

From the superpotential  $W_{dual} = (M'' q'' \tilde{q}'' + m'' M'') + X' g q'' + \tilde{X}' \tilde{q}'' \tilde{g} + \Phi' g \tilde{g}$ ,  $q'' \tilde{q}''$  has rank  $\tilde{N}''_c$  while  $m''$  has a rank  $N''_f$ . The derivative of the superpotential  $W_{dual}$  with respect to  $M''$  cannot be satisfied if the rank  $N''_f$  exceeds  $\tilde{N}''_c$  and the supersymmetry is broken. The classical moduli space of vacua can be obtained from F-term equations. Then the solutions can be written as

$$\begin{aligned} \langle q'' \rangle &= \begin{pmatrix} \sqrt{m''} e^\phi \mathbf{1}_{\tilde{N}''_c} \\ 0 \end{pmatrix}, \langle \tilde{q}'' \rangle = \begin{pmatrix} \sqrt{m''} e^{-\phi} \mathbf{1}_{\tilde{N}''_c} & 0 \end{pmatrix}, \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M''_0 \mathbf{1}_{N''_f - \tilde{N}''_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

It turns out that states are stable by realizing the mass of  $m_{M''_0}^2$  positive, by expanding the fields around the vacua.

Then the gauge group and matter contents we consider are summarized as follows:

	gauge group :	$Sp(N_c) \times SU(N'_c) \times SU(\tilde{N}''_c)$
matter :	$Q_f$	$(\square, \mathbf{1}, \mathbf{1}) \quad (f = 1, \dots, 2N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$q''_{f''} \oplus \tilde{q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\square, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X'_{n''} \equiv) GQ'' \oplus \tilde{G}\tilde{Q}'' (\equiv \tilde{X}'_{n''})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n'', \tilde{n}'' = 1, \dots, N''_f)$
	$(M''_{f'', \tilde{g}''} \equiv) Q''\tilde{Q}''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f'', \tilde{g}'' = 1, \dots, N''_f)$
	$(\Phi' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The nonsupersymmetric minimal energy brane configuration Figure 18B with a replacement of  $N''_f$  D6-branes with NS5'-brane(neglecting the  $NS5'_L$ -brane,  $N_f$  D6-branes,  $N'_f$  D6-branes, and  $N_c$  D4-branes) leads to the Figure 14B of [27].

At nonzero string coupling constant, the NS5-branes bend due to their interactions with the D4-branes and D6-branes. Then the behavior of the supersymmetric M5-brane curves can be summarized as follows:

1.  $v \rightarrow \infty$  limit implies

$$\begin{aligned} w &\rightarrow 0, \quad y \sim v^{\tilde{N}''_c} + \dots \quad \overline{NS5} \text{ asymptotic region,} \\ w &\rightarrow 0, \quad y \sim v^{-\tilde{N}''_c + 2N''_f + 2N'_f + 2N_f - 4} + \dots \quad NS5 \text{ asymptotic region.} \end{aligned}$$

2.  $w \rightarrow \infty$  limit implies

$$\begin{aligned} v &\rightarrow +m'', \quad y \sim w^{N'_c + N''_f + 2N_f + N'_f - 2N_c - 4} + \dots \quad NS5'_L \text{ asymptotic region,} \\ v &\rightarrow +m'', \quad y \sim w^{N''_f + 2N_f + 2N'_f - N'_c + \tilde{N}''_c - 4} + \dots \quad NS5'_R \text{ asymptotic region,} \\ v &\rightarrow -m'', \quad y \sim w^{N''_f + N'_f + 2N_c - N'_c} + \dots \quad \overline{NS5}'_L \text{ asymptotic region,} \\ v &\rightarrow -m'', \quad y \sim w^{N'_c + N''_f - \tilde{N}''_c} + \dots \quad \overline{NS5}'_R \text{ asymptotic region.} \end{aligned}$$

## 5.6 Magnetic theories for the multiple product gauge groups

Now one can generalize the method for the triple product gauge groups to the finite  $n$ -multiple product gauge groups characterized by

$$Sp(N_{c,1}) \times SU(N_{c,2}) \cdots \times SU(N_{c,n})$$

with the matter, the  $(n-1)$  bifundamentals  $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_{n-1}, \bar{\square}_n)$ , their complex conjugate  $(n-1)$  fields  $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \bar{\square}_{n-1}, \square_n)$ , linking the gauge groups together,  $(n-1)$ -fundamentals  $(\mathbf{1}_1, \square_2, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \square_n)$ , and  $(n-1)$ -antifundamentals  $(\mathbf{1}_1, \bar{\square}_2, \dots, \mathbf{1}_n), \dots$ , and  $(\mathbf{1}_1, \dots, \mathbf{1}, \bar{\square}_n)$  and one fundamental in the representation  $(\square_1, \dots, \mathbf{1}_n)$ . Then the mass-deformed superpotential can be written as  $W_{elec} = m_1 Q_1 \bar{Q}_1 + \sum_{i=2}^n m_i Q_i \bar{Q}_i$ . The brane configuration can be constructed from Figure 15 by adding  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes to the right of  $NS5'_R$ -brane (and its mirrors) leading to the fact that any two neighboring NS-branes should be perpendicular to each other.

There exist  $(2n-3)$  magnetic theories and they can be classified as follows.

- When the dual magnetic gauge group is  $Sp(\tilde{N}_{c,1})$

There is no nonsupersymmetric meta-stable brane configuration.

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,2})$

When the Seiberg dual is taken for the second gauge group factor by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 5.3. The gauge group is given by

$$Sp(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + 2N_{c,1} + N_{c,3} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 16. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + g_2 \tilde{X}_3 \tilde{q}_2 + \tilde{g}_2 q_2 X_3 + \Phi_3 g_2 \tilde{g}_2 \right) + m_2 M_2.$$

When the Seiberg dual is taken for the second gauge group factor with different brane motion by assuming that  $\Lambda_2 \gg \Lambda_j$  where  $j = 1, 3, \dots, n$ , one follows the procedure given in the subsection 5.4. The gauge group is given by

$$Sp(N_{c,1}) \times SU(\tilde{N}_{c,2} \equiv N_{f,2} + N_{c,3} + 2N_{c,1} - N_{c,2}) \times SU(N_{c,3}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-i-1)$  NS-branes,  $(n-i-1)$  sets of D6-branes and  $(n-i-1)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 17. The magnetic superpotential can be written as

$$W_{dual} = \left( M_2 q_2 \tilde{q}_2 + f_1 X_1 q_2 + \tilde{f}_1 \tilde{q}_2 \tilde{X}_1 + \Phi_1 f_1 \tilde{f}_1 + Q_1 \Phi_1 Q_1 \right) + m_2 M_2.$$

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,i})$  where  $3 \leq i \leq n-1$

When the Seiberg dual is taken for the middle gauge group factor by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i-1, i+1, \dots, n$ , one follows the procedure given in the subsection 2.3 of [18]. The gauge group is given by

$$Sp(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i-2)$  NS-branes,  $(i-2)$  sets of D6-branes and  $(i-2)$  sets of D4-branes are present between O6-plane and the NS5-brane in Figure 16 and the extra  $(n-i-1)$  NS-branes,  $(n-i-1)$  sets of D6-branes and  $(n-i-1)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 16. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + g_i \tilde{X}_{i+1} \tilde{q}_i + \tilde{g}_i q_i X_{i+1} + \Phi_{i+1} g_i \tilde{g}_i \right) + m_i M_i.$$

When the Seiberg dual is taken for the middle gauge group factor with different brane motion by assuming that  $\Lambda_i \gg \Lambda_j$  where  $j = 1, 2, \dots, i-1, i+1, \dots, n$ , one follows the procedure given in the subsection 2.4 of [18]. The gauge group is given by

$$Sp(N_{c,1}) \times \dots \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times \dots \times SU(N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(i-2)$  NS-branes,  $(i-2)$  sets of D6-branes and  $(i-2)$  sets of D4-branes are present between O6-plane and the NS5-brane in Figure 17 and the extra  $(n-i-1)$  NS-branes,  $(n-i-1)$  sets of D6-branes and  $(n-i-1)$  sets of D4-branes are present at the right hand side of the  $NS5'_R$ -brane of Figure 17. The magnetic superpotential can be written as

$$W_{dual} = \left( M_i q_i \tilde{q}_i + f_{i-1} X_{i-1} q_i + \tilde{f}_{i-1} \tilde{q}_i \tilde{X}_{i-1} + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \right) + m_i M_i.$$

- When the dual magnetic gauge group is  $SU(\tilde{N}_{c,n})$

When the Seiberg dual is taken for the last gauge group factor by assuming that  $\Lambda_n \gg \Lambda_i$  where  $i = 1, 2, \dots, (n-1)$ , one follows the procedure given in the subsection 5.5. The gauge group is given by

$$Sp(N_{c,1}) \times \dots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra  $(n-3)$  NS-branes,  $(n-3)$  sets of D6-branes and  $(n-3)$  sets of D4-branes are present between the O6-plane and the  $NS5'_L$ -brane of Figure 18. The magnetic superpotential can be written as

$$W_{dual} = \left( M_n q_n \tilde{q}_n + g_{n-1} X_{n-1} q_n + \tilde{g}_{n-1} \tilde{q}_n \tilde{X}_{n-1} + \Phi_{n-1} g_{n-1} \tilde{g}_{n-1} \right) + m_n M_n.$$



## 6 Conclusions and outlook

The meta-stable brane configurations we have found are summarized by Figures 2B, 3B, 4B, and 5B for the first gauge theory described in section 2, by Figures 7B, 8B, 9B, and 10B for the second gauge theory given in section 3, by Figures 12B, 13B, and 14B for the third gauge theory in section 4, and by Figures 16B, 17B, and 18B for the last gauge theory described in section 5.

Some observations are found as follows:

- The nonsupersymmetric minimal energy brane configuration Figure 2B with a replacement  $N_f$  D6-branes by the NS5'-brane(neglecting the  $NS5'_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes and  $N_f'$  D6-branes) leads to the Figure 5B of [23] with a rotation of NS5'-brane by  $\frac{\pi}{2}$  angle. The Figure 3B with a replacement  $N_f'$  D6-branes by the NS5'-brane(neglecting the  $NS5'_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes and  $N_f$  D6-branes) becomes the Figure 2B of [23] with a rotation of NS5'-brane by  $\frac{\pi}{2}$  angle. Moreover, the Figure 5B with a replacement  $N_f''$  D6-branes by the NS5'-brane(neglecting the  $NS5'_L$ -brane,  $N_f$  D6-branes and  $N_c$  D4-branes and  $N_f'$  D6-branes) turns out to be the Figure 4B of [23].

- The nonsupersymmetric minimal energy brane configuration Figure 7B with a replacement  $N_f''$  D6-branes by the NS5'-brane(neglecting the  $NS5_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes and  $N_f'$  D6-branes) leads to the Figure 7B of [23] with a rotation of NS5-brane by  $\frac{\pi}{2}$  angle and the Figure 8B with a replacement  $N_f'$  D6-branes by the NS5'-brane(neglecting the  $NS5_R$ -brane,  $N_f''$  D6-branes and  $N_c''$  D4-branes and  $N_f$  D6-branes) reduces to the Figure 8B of [23].

- The nonsupersymmetric minimal energy brane configuration Figure 12B with a replacement  $N_f'$  D6-branes by the NS5'-brane(neglecting the  $NS5_R$ -brane,  $N_f''$  D6-branes,  $N_f$  D6-branes, and  $N_c''$  D4-branes) becomes the Figure 15B of [27].

- The nonsupersymmetric minimal energy brane configuration Figure 16B with a replacement  $N_f'$  D6-branes by the NS5'-brane(neglecting the  $NS5'_R$ -brane,  $N_f''$  D6-branes,  $N_c''$  D4-branes and  $N_f$  D6-branes) leads to the Figure 12B of [27] while the nonsupersymmetric minimal energy brane configuration Figure 18B with a replacement of  $N_f''$  D6-branes with NS5'-brane(neglecting the  $NS5'_L$ -brane,  $N_f$  D6-branes,  $N_f'$  D6-branes, and  $N_c$  D4-branes) gives rise to the Figure 14B of [27].

It would be very interesting to find out how the meta-stable brane configurations from type IIA string theory including the present work are related to some brane configurations found in recent works [28]-[65] where some of them are described in the type IIB string theory.

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